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Smooth Transition GARCH Models in Forecasting Non-Linear Economic Time Series Data

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Abstract: The need to capture the heterogeneous and volatility nature of both financial and economic time series theory and modeling their behavior in practical work have stimulated interest in the empirical modeling of variances which forms the basis for this study. In the study we augmented GARCH models with smooth transition model by dropping the assumption of autoregression of the model; necessary theoretical frame work was derived and properties of the new model established and illustrated with foreign exchange rate data from Federal Republic of Nigeria (Naira), Great Britain (Pound), Botswana (Pula) and Japanese (Yen) against United States of America (Dollar). The smooth transition GARCH model is better than the classical GARCH model as there were reduction in the variances of the augmented model; this claim is confirmed by the empirical illustration with foreign exchange data. Within the group of smooth transition GARCH model, Logistic Smooth Transition is adjudged the best as it produced the least variance.

Key words: GARCH models; ST-GARCH models; ET-GARCH; EST-GARCH; LST-GARCH; Foreign exchange data

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1. INTRODUCTION

Financial time series experts have discovered evidence of occasional sudden breaks in many economic time series. For example, currency exchange rates often move suddenly as governments devalue due to speculative pressure and deteriorating economic conditions. Commodity prices, such as oil, change in response to shocks from exogenous geopolitical events or supply disruptions due to weather related catastrophes like hurricane Katrina, and financial markets can shift abruptly in response to financial crises [1]. Perhaps the best example of this is economic growth, where the rate of growth of the economy alternates between periods of high growth (economic expansion) and periods of declining or negative growth (recession). It is an established fact that Generalized autoregressive model cannot capture nonlinearity in economic and financial series adequately; [2] utilized GARCH to model stock market indices and concluded that the model fails to capture the statistical structure of the market returns series for all the countries economies investigated. [3] employed the Hinich portmanteau bicorrelation test to determine the adequacy of GARCH model for eight Asian stock markets and equally concluded that GARCH is not suitable. Due to the inadequacy of GARCH model there is need to augment it with non-linear models, in this paper we combine GARCH model with Smooth transition model, the various transition models used are Exponential transition (ET), Exponential smooth transition (EST) and Logistic smooth transition (LST) models. The smooth transition is an extension of the regime switching model that allows intermediate states or regimes. The idea of smooth transition was proposed to allow a more gradual change for the parameter of transition. Thus, the ST-GARCH allows enriching the class of GARCH models, through asymmetry, or the leverage effect, that is the difference in the volatility response to positive and negative return shocks [4]. This model emphasizes the nonlinearities in the conditional volatility equation. Although a variety of models were already presented in the literature in order to explore different forms of nonlinearity, Hagerud (1997) [5] affirms that the ST-GARCH model presents new characteristics, very advantageous for the modeling of the volatility. So also this model provides more flexibility in the transition mechanism of the conditional volatility. Contrary to the traditional threshold models, that allow only two volatility regimes (a low volatility regime and a high volatility regime), the ST-GARCH model offers the possibility of intermediate regimes and allows the introduction of a smoother transition mechanism in the GARCH specification [6]. In this paper, the Smooth Transition Generalized Autoregressive Conditional Heteroscedastic (ST-GARCH) model is considered, where the possibility of intermediate regimes is modeled with the introduction of a smooth transition mechanism in a Generalized Autoregressive Conditional Heteroscedastic (GARCH) specification. The transition functions utilized are logistic (the Logistic Smooth Transition GARCH (LST-GARCH) model) Exponential (the Exponential Transition GARCH (ET-GARCH) and the Exponential Smooth Transition GARCH (EST-GARCH) model). An important characteristic of the LST-GARCH model is that it highlights the asymmetric effect of unanticipated shocks on the conditional volatility. On the other side, the ET-GARCH and EST-GARCH model allow the dynamics of the conditional variance to be independent of the sign of past news. Indeed, this model allows us to highlight the size effect of the shocks, so that small and big shocks have separate effects.

The major advantage of this model is that it challenges the assumption in the basic GARCH model that the model parameters are constant over time, by introducing nonlinearities and regime changes for the conditional volatility. Other advantages are that this model allows us to highlight a significant characteristic of the financial series: the asymmetric effect in the conditional volatility following surprise shocks.

The remaining part of this paper is organized as follows: section 2-5 covers the Specification of transition models with GARCH, estimation of the parameters of ST-GARCH models, efficiency of ST-GARCH models with respect to GARCH model, section 6, Empirical analysis, estimation of γ and c estimation of ET-GARCH, EST-GARCH and LST-GARCH for all series section 7 Empirical comparisons of models section 8 Conclusion.

2. EXPONENTIAL TRANSITION-GARCH MODEL (ET-GARCH)

As earlier defined the ET model is of the form

$$G_A(\gamma, c; y_{t-j}) = \exp\left\{-\gamma y_{t-1}^2\right\}, \gamma > 0 \tag{1}$$

The function G_A is symmetric around zero, where it obtains the value one, and $G_A(\gamma, c; y_{t-j}) \to 0$ as $\left|y_{t\text{ - }j}\right| \to \infty$. In case of ET model, parameter vectors do not contain intercept terms, i.e., Equation (1) indicates that the model can be interpreted as a linear autoregressive model with stochastic time-varying coefficients $\varphi + \theta G_A(\gamma, c; y_{t-j})$. When $\gamma \to 0$ or $\gamma \to \infty$, the model becomes linear. In the latter case, $G_A(\gamma, c; y_{t-j}) = 0$, except when $y_{t-j} = 0$. Prominent feature of this model is that it can generate limit cycles by itself.

Let $G_A(\gamma, c; y_{t-j}) =$ $\exp\left\{-\gamma y_{t-j}^2\right\}$, $\gamma > 0$

From the usual GARCH models we have

$$y_t = \sigma_t \varepsilon_t \tag{2}$$

where

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{3}$$

$$E\left(y_{t-j}^2\right) = \frac{\alpha_0}{1 - \sum\limits_{i=1}^{p} (\alpha_i + \beta_j)} \tag{4}$$

Square of (2) with delay parameter gives $y_{t-j}^2 = \sigma_{t-j}^2 \varepsilon_{t-j}^2$ and therefore the ET-GARCH model specification will be

$$G_A = \exp\left(-\gamma\left(\sigma_{t-j}^2 \varepsilon_{t-j}^2\right)\right) \qquad 0 < \gamma < 1 \tag{5}$$

Now, without loss of generality, we can derive the log ET-GARCH as

$$G_A^* = \log G_A = -\gamma y_{t-j}^2 \tag{6}$$

and y_{t-j}^2 follows the conventional GARCH form with as delay parameter.

The mean of $G_A^\bullet$ could be derived from our earlier expressions in chapter four as follows:

By taking the expected value of Equation (5), we have $E\left(G_A^*\right) = -\gamma E\left(y_{t-j}^2\right)$ and Equation (4), we have

$$E\left(G_A^*\right) = \frac{-\gamma\alpha_0}{1 - \sum\limits_{j=1}^{p}\left(\alpha_i + \beta_j\right)} = \frac{\gamma\alpha_0}{\sum\limits_{j=1}^{p}\left(\alpha_i + \beta_j\right) - 1} \tag{7}$$

Equation (7) gives the mean of log ET-GARCH model. The variance of log ET-GARCH is derived as follows:

From Equation (3), we have $Var.\left(G_A^*\right) = \gamma^2 Var.\left(y_{t-j}^2\right) = \gamma^2 Var.\left(\sigma_{t-j}^2 \varepsilon_{t-j}^2\right)$.

Since σ_{t-j}^2 and ε_{t-j}^2 are independent and uncorrelated random variables, then $Var.\left(G_A^*\right) = \gamma^2 Var.\left(\sigma_{t-j}^2\right) Var.\left(\varepsilon_{t-j}^2\right) = \gamma^2 Var.\left(\sigma_{t-j}^2\right)$, since $Var.\left(\varepsilon_{t-j}^2\right) = 1$.

If σ_{t-j}^2 are independent and identically distributed then we have

$$Var.\left(G_A^*\right) = \gamma^2 \left\{\frac{2\sigma^4}{n-1}\right\} \tag{8}$$

where σ^2 is the variance of variance equation for GARCH model fitted.

3. EXPONENTIAL SMOOTH TRANSITION GARCH MODEL (EST-GARCH)

The ET model may be generalized by allowing an intercept $\phi_0 \neq 0$ or $\theta_0 \neq 0$ or both. The purpose of the generalization is to make the ET model location invariant. Thus, the function g_E becomes

$$g_E\left(\gamma, c; y_t\right) = 1 - \exp\left\{-\gamma(y_t - c)^2\right\}, \quad \gamma \succ 0 \tag{9}$$

Terasvirta (1994) called this model as Exponential smooth transition autoregressive (ESTAR) model and discussed procedure for estimation of parameters. It has the property that the minimum value of the transition function equals zero. It has been successfully used to model macroeconomic series, such as strongly fluctuating inflation series (Baharumshah and Liew, 2006).

Let $g_E = 1 - \exp\left\{-\gamma(y_t - c)^2\right\} \gamma > 0$, the Exponential smooth transition GARCH model specification is $g_E = 1 - \exp\left\{-\gamma(y_t - c)^2\right\}$, therefore,

$$G_E = 1 - g_E = \exp\left\{-\gamma(y_t - c)^2\right\} = \exp\left\{-\gamma(\sigma_t\varepsilon_t - c)^2\right\} \tag{10}$$

Such that by taking the logarithm gives

$$G_E^\bullet = \log G_E = -\gamma(\sigma_t\varepsilon_t - c)^2 = -\gamma\sigma_t^2\varepsilon_t^2 + 2\gamma c\sigma_t\varepsilon_t + \gamma c^2 \tag{11}$$

Equation (10) gives the log EST-GARCH model. The mean of G_E^* is derived as follows: taking the expectation of (10) gives

$$E\left(G_E^\bullet\right) = -\gamma E\left(\sigma_t^2\varepsilon_t^2\right) + 2\gamma c E\left(\sigma_t\right) E\left(\varepsilon_t\right) + \gamma c^2 = -\gamma E\left(y_t^2\right) + \gamma c^2$$

Using Equation (4) in the last expression we have

$$E\left(G_E^*\right)=\frac{\gamma\alpha_0}{\sum\limits_{j=1}^{p}\left(\alpha_i+\beta_j\right)-1}+\gamma c^2=\gamma\left[\frac{\alpha_0}{\sum\limits_{j=1}^{p}\left(\alpha_i+\beta_j\right)-1}+c^2\right] \tag{12}$$

Equation (5) gives the mean of the log EST-GARCH. The variance of log EST-GARCH defined in Equation (10) is

$$\begin{aligned} Var(G_E^*) &= \gamma^2 Var(\sigma_t^2)+4\gamma^2c^2 Var\left(\sigma_t\right) \\ &= \gamma^2\left[\frac{2\sigma^4}{n-1}\right]+4\gamma^2c^2\left[\frac{\sqrt{2}\sigma^2}{\sqrt{n-1}}\right] \\ &= 2\gamma^2\sigma^2\left[\frac{\sigma^2}{n-1}+\frac{2c^2\sqrt{2}}{\sqrt{n-1}}\right] \\ &= 2\gamma^2\sigma^2\left[\frac{\sigma^2+2c^2\sqrt{n-1}\sqrt{2}}{n-1}\right] \end{aligned} \tag{13}$$

4. LOGISTIC SMOOTH TRANSITION GARCH MODEL (LST-GARCH)

This model is defined by Equation (14), where the transition function is the logistic function

$$g_L\left(\gamma,c;y_{t-j}\right)=\left\{1+\exp\left\{-\gamma\prod_{i=1}^{k}\left(y_{t-j}-c\right)\right\}\right\}^{-1},\quad \gamma\succ 0 \tag{14}$$

Let $g_L=\dfrac{1}{1+\exp\left\{-\gamma\prod\limits_{i=1}^{k}\left(\sigma_t\varepsilon_t-c\right)\right\}}$, by taking inverse and logarithm with simple algebraic manipulations gives

$$\frac{1}{g_L}-1=\exp\left\{-\gamma\prod_{i=1}^{k}\left(\sigma_t\varepsilon_t-c\right)\right\}=\log\left[\exp\left\{-\gamma\prod_{i=1}^{k}\left(\sigma_t\varepsilon_t-c\right)\right\}\right]$$

Since $y_t=\sigma_t\varepsilon_t$, then

$$G_L^*=\exp\left\{-\gamma\left(y_t-c\right)\right\} \tag{15}$$

From Equation (15), we could derive the mean of transformed LT-GARCH model as

$$\begin{aligned} E\left(G_L^*\right) &= -\gamma\prod_{t=1}^{k}\left(y_t-c\right) \\ &= -\gamma\left[\left(E(y_1-c\right)\left(E(y_2-c\right)...\left(E(y_k-c\right)\right] \\ &= -\gamma\left[\left(E\left(y_1\right)-c\right)\left(E\left(y_2\right)-c\right)...\left(E\left(y_k\right)-c\right)\right] \\ &= -\gamma c^k \quad \text{since} \quad E\left(y_t\right)=0 \quad \forall i \\ &= -\gamma c^2 \quad \text{where} \quad k=2. \end{aligned} \tag{16}$$

$$G_L^* = \frac{1}{g_L} - 1(\text{ a transformed LTGM})$$

The variance of G_L^* is derived as follows:

$$\begin{aligned}
Var\left(G_L^*\right) &= \gamma^2 Var\left\{\prod_{t=1}^{k}\left(y_t - c\right)\right\} \\
&= \gamma^2 Var\prod_{t=1}^{k}\left(y_t - c\right) \\
&= \gamma^2\prod_{t=1}^{k} Var\left(y_t\right) \\
&= \gamma^2\prod_{t=1}^{k} Var\left(\sigma_t\varepsilon_t\right) = \gamma^2 kVar\left(\sigma_t\right) \\
&= k\gamma^2\sigma^2\sqrt{\frac{2}{n-1}} \\
&= 2\gamma^2\sigma^2\sqrt{\frac{2}{n-1}} \quad \text{since} \quad k = 2
\end{aligned} \tag{17}$$

Table 1
Summarized Theoretical Derivation Tables of the Means and Variances of All the Models Used in This Paper with GARCH Models

Statistic /Model	GM	ET-GARCH	EST-GARCH	LST-GARCH
Mean	$\sigma_t\varepsilon_t$	$= \frac{\gamma\alpha_0}{\sum_{j=1}^{p}(\alpha_i+\beta_j)-1}$	$= \gamma\left[\frac{\alpha_0}{\sum_{j=1}^{p}(\alpha_i+\beta_j)-1} + c^2\right]$	$-\gamma c^2$
Variance	$\frac{\alpha_0}{1-\sum_{i=1}^{p}(\alpha_i+\beta_j)}$	$\gamma^2\left\{\frac{2\sigma^4}{n-1}\right\}$	$= 2\gamma^2\sigma^2\left[\frac{\sigma^2+2c^2\sqrt{n-1}\sqrt{2}}{n-1}\right]$	$= 2\gamma^2\sigma^2\sqrt{\frac{2}{n-1}}$

5. EFFICIENCY OF ST-GARCH MODELS WITH RESPECT TO GARCH MODEL

To compare the efficiency of the ST-GARCH with GARCH, we relate the variances of ST-GARCH to that of classical GARCH we have

(i). $$\frac{Var\left(y_{t(ET-GARCH)}\right)}{Var\left(y_{t(GM)}\right)} = \frac{2\gamma^2\sigma^4\left(1-\sum\left(\alpha_i+\beta_i\right)\right)}{\alpha_0\left(n-1\right)}$$

if $2\gamma^2\sigma^4\left(1-\sum\left(\alpha_i+\beta_i\right)\right) < \alpha_0(n-1)$, then ET-GARCH is more efficient than GARCH.

(ii). $$\frac{Var\left(y_{t(EST-GARCH)}\right)}{Var\left(y_{t(GM)}\right)} = \frac{2\gamma^2\sigma^2\left(\sigma^2+2c^2\sqrt{n-1}\sqrt{2}\right)1-\sum_{i=1}^{p}\left(\alpha_i+\beta_i\right)}{\alpha_0\left(n-1\right)}$$

So if $2\gamma^2\sigma^2\left(\sigma^2+2c^2\sqrt{n-1}\sqrt{2}\right)1-\sum_{i=1}^{p}(\alpha_i+\beta_i)<\alpha_0(n-1)$, then EST-GARCH is more efficient than GARCH.

(iii). $$\frac{Var\left(y_{t(LST-GARCH)}\right)}{Var\left(y_{t(GM)}\right)}=\frac{2\gamma^2\sigma^2\sqrt{2(n-1)}\left(1-\sum_{i=1}^{p}(\alpha_i+\beta_i)\right)}{(n-1)\,\alpha_0}$$

So if $2\gamma^2\sigma^2\sqrt{2(n-1)}\left(1-\sum_{i=1}^{p}(\alpha_i+\beta_i)\right)<\alpha_0(n-1)$, then LST-GARCH is more efficient than GARCH.

The mathematical expression in all equations derived (i-iii) for the models are trivial and could be best appreciated by using an empirical approach, but if $\alpha_0(n-1)$ is greater than the quantities in the numerator then the GARCH model is less efficient.

6. EMPIRICAL RESULTS/DATA ANALYSIS WITH EXCHANGE RATE DATA

This section examines the empirical results obtained for Smooth transition models with GARCH (ST-GARCH) for four sets of exchange rates data namely British (Pounds), Japanese (Yen), Nigerian (Naira) and Batswana (Pula) against American (Dollar). Here the Parameters of Exponential transition GARCH models (ET-GARCH), Exponential smooth transition GARCH (EST-GARCH) and Logistic transition GARCH (LST-GARCH) models were obtained using the derived equations for all the series.

The following values of variances were obtained for classical GARCH models:

Table 2
The GARCH Model Fitted for All Series

Series		**Coefficient (S.E)**		**Model variance**
	α_0	α_1	β_1	
Naira	3.85802 (0.34152)	1.16179 (0.52198)	-0.99980 (0.00016)	4949.20411
Pound	0.00017 (0.00005)	0.97219 (0.23370)	-0.00024 (0.02657)	0.65816
Pula	0.047613 (0.10084)	1.90362 (0.30959)	-0.91061 (0.01367)	2.14441
Yen	0.67948 (0.26140)	1.00818 (0.26247)	-0.03111 (0.12408)	5461.26025

6.1. Estimation of γ and c

Starting values needed for the nonlinear optimization algorithm can be obtained using two dimensional grid search over γ and c, and select those that give smallest estimator for the residual variance. The two dimensional grid give three possible values as tabulated below:

Table 3
Values of Grid of c

Series	I	II	III
Naira	$0.35^{\bullet}$	155.76	30
Pound	$0.48^{\bullet}$	2.42	30
Pula	$0.74^{\bullet}$	7.97	30
Yen	$0.74^{\bullet}$	7.97	30

Table 4
Values of Grid of γ

Series	I	II	III
Naira	$0.50^{\bullet}$	10.00	30
Pound	$0.50^{\bullet}$	10.00	30
Pula	$0.50^{\bullet}$	10.00	30
Yen	$0.50^{\bullet}$	10.00	30

In the tables 3 and 4 above, all the asterisk values are selected because they have minimum values and are subsequently used in equations (8), (13) and (18).

We can now illustrate the empirical implication of these theories as follows:

6.2. Variances of Transition GARCH Models (ST- GARCH) with Classical GARCH Model

Using equations (8), (13) and (18) the variances of all the series for Smooth transition GARCH models (ET-GARCH, EST-GARCH and LT-GARCH) with classical GARCH model are computed and shown/displayed in Table 5.

Table 5 shows the variances of GARCH model and those of ST-GARCH models (ET-GARCH, EST-GARCH and LST-GARCH). The superiority of Logistic Smooth transition model within the group of smooth transition was asserted here as this model gave us the minimum variances for all the series under study, this is followed by EST-GARCH and ET-GARCH in that order as seen in Table 5 below:

Table 5
Variances of ST-GARCH Models with Classical GARCH Model

Series	G.M	ET-GARCH	EST-GARCH	LST-GARCH
Naira	4949.2041	1292.9	1290.014	5.3965
Pound	0.6582	0.0066	0.0002	0.0006
Pula	2.1444	0.0840	0.0242	0.0740
Yen	5461.2603	1831.9096	1817.151	6.4046

6.3. *Efficiencies of all Smooth Transition GARCH Models (ST- GARCH) with GARCH Model*

Efficiencies of all transition models were compared as shown in Table 6, where Logistic Smooth transition model was adjudged the model that produced the most efficient values among the models under study for all series; this is followed by EST-GARCH.

Table 6
The Efficiencies of all Smooth Transition Models with GARCH Model

Series	ET-GARCH	EST-GARCH	LST-GARCH
Naira	0.2613	0.2607	0.0011
Pound	0.0090	0.003	0.0009
Pula	0.0390	0.0113	0.0035
Yen	0.3355	0.3327	0.0012

7. CONCLUSION

The variances of all ST-GARCH models with GARCH as displayed in Table 5 shows that all ST-GARCH outperformed the classical GARCH model, however, the LST-GARCH performed best, followed by the performances of LST-GARCH and ET-GARCH in that order. The implication is that the use of LSTCGARCH produces the best result; however EST-GARCH and ET-GARCH may be utilized in some occasions. But LST-GARCH produces optimal result.

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