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Reduced-Order Projective Synchronization of Hyper-Chaotic LÜ System and Chen System

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Abstract: By selecting non-zero constant as a scaling factor, we design a reduced-order projective synchronization scheme for synchronizing the fourth-order hyper-chaotic Lü system and the third-order chaotic Chen system. To this end, a nonlinear synchronization controller is constructed. Finally, some numerical simulations are given to illustrate the feasibility and effectiveness of the proposed synchronization scheme in this paper.

Key words: Reduced-order; Projective synchronization; Hyper-chaotic Lü system; Chen system

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1. INTRODUCTION

Our natural world is undoubtedly nonlinear. Hence, chaos is inevitable in our lives, even though it may not be seen with the naked eye. Many literatures showed that nonlinear system can display complex behaviors including bifurcations, chaos, hyper-chaos, and so on. In the past twenty years, chaos synchronization has been extensively studied not only for its importance in theory but also for its prospective applications in many areas such as biological systems, information processing, secure communications [1], etc. At present a lot of synchronization schemes have been proposed, for example, complete synchronization (CS) [2], anti-synchronization (AS) [3], phase synchronization (PS) [4], generalized synchronization (GS) [5], etc. However, in these synchronization schemes the drive system and the response system always have same order. Now the synchronization of chaotic systems with different order has received less attention [6–9]. In fact, the synchronization phenomena of chaotic systems with different order are the more common form. In the case of thalamic neurons, for instance, such a problem is reasonable if their order is different from the one of the hippocampal neurons [9]. The synchronization between heart and lung is another example. One can observe that both, circulatory and respiratory systems, behave in synchronous way. In this paper we will design a synchronization scheme to realize projective synchronization between fourth-order hyper-chaotic Lü system and third-order Chen system.

2. SYNCHRONIZATION SCHEME DESIGN

In this section, a reduced-order projective synchronization scheme for synchronizing hyper-chaotic Lü system and Chen system is designed. The hyper-chaotic Lü system is given by [9,10]

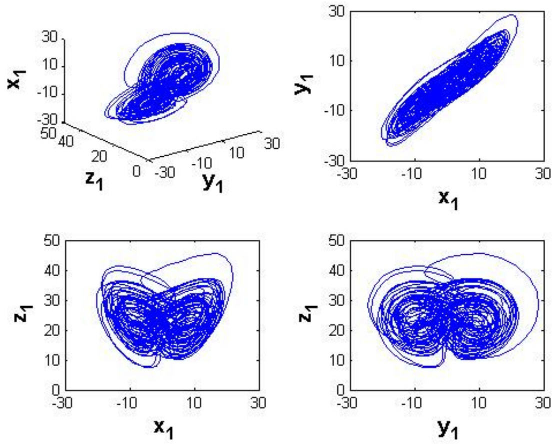

Figure 1
Projection of Hyper-Chaotic LÜ Attractor

$$\begin{cases} \dot{x}_1 = a_1(y_1 - x_1) + w_1 \\ \dot{y}_1 = -x_1 z_1 + c_1 y_1 \\ \dot{z}_1 = x_1 y_1 - b_1 z_1 \\ \dot{w}_1 = x_1 z_1 + r_1 w_1 \end{cases}, \tag{1}$$

where x_1, y_1, z_1 and w_1 are state variables, and a_1, b_1, c_1 and r_1 are model parameters. System (1) has periodic orbit when $a_1 = 36$, $b_1 = 3$, $c_1 = 20$, $-1.03 \leq r_1 \leq -0.46$, and system (1) appears chaotic behavior when $a_1 = 36$, $b_1 = 3$, $c_1 = 20$, $-0.46 \leq r_1 \leq -0.35$ and system (1) has hyper-chaotic attractor when $a_1 = 36$, $b_1 = 3$, $c_1 = 20$, $-0.35 \leq r_1 \leq 1.3$. Figure 1 shows the projections of the attractor of hyper-chaotic Lü system with the parameter values $a_1 = 36$, $b_1 = 3$, $c_1 = 20$ and $r_1 = 1$.

The dynamical equations of chaotic Chen system is described by [9]

$$\begin{cases} \dot{x}_2 = a_2(y_2 - x_2) + u_1 \\ \dot{y}_2 = (c_2 - a_2)x_2 - x_2 z_2 + c_2 y_2 + u_2 \\ \dot{z}_2 = x_2 y_2 - b_2 z_2 + u_3 \end{cases} \tag{2}$$

where x_2, y_2 and z_2 are state variables, and a_2, b_2 and c_2 are model parameters. Notations u_1, u_2 and u_3 are three control functions to be designed. When $a_2 = 35$, $c_2 = 28$, $b_2 = 3$, system (2) has a chaotic attractor. The chaotic attractor of Chen system and its projections have been shown in Figure 2 for the given parameter values $a_2 = 35$, $c_2 = 28$, $b_2 = 3$ and $u_1 = 0$, $u_2 = 0$, $u_3 = 0$.

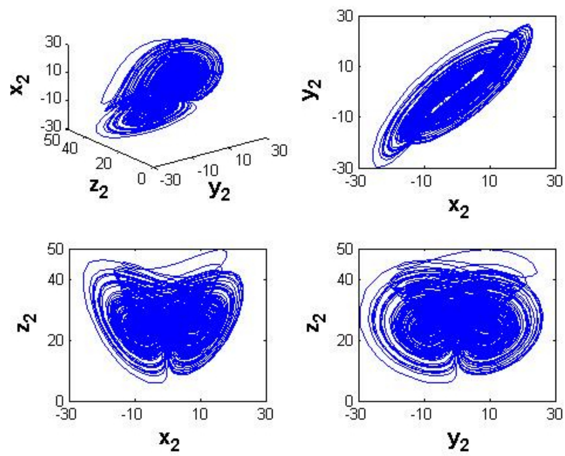

Figure 2
Projection of Chaotic Chen Attractor

In the following text, we will design reduced-order projective synchronization scheme between Chen chaotic system (2) and the projective subsystem which is constructed by the first three equations of hyper-chaotic Lü system. Suppose that m is a non-zero constant, and let $e_1 = x_2 - m \cdot x_1$, $e_2 = y_2 - m \cdot y_1$ and $e_3 = z_2 - m \cdot z_1$ denoting synchronization errors.

Define 1. Systems (1) and (2) are referred to as achieving reduced-order projective synchronization if and only if the three following equalities satisfy simultaneously:

$$\lim_{t\to\infty} e_1(t) = \lim_{t\to\infty} |x_2(t) - m \cdot x_1(t)| = 0,$$
$$\lim_{t\to\infty} e_2(t) = \lim_{t\to\infty} |y_2(t) - m \cdot y_1(t)| = 0,$$
$$\lim_{t\to\infty} e_3(t) = \lim_{t\to\infty} |z_2(t) - m \cdot z_1(t)| = 0.$$

By using systems (1) and (2), the error dynamical system can be obtained as below:

$$\begin{cases} \dot{e}_1 = a_2 y_2 - a_2 x_2 - m a_1 y_1 + m a_1 x_1 - m w_1 + u_1 \\ \dot{e}_1 = c_2 x_2 - a_2 x_2 - x_2 z_2 + c_2 y_2 + m x_1 z_1 - m c_1 y_1 + u_2 \\ \dot{e}_1 = x_2 y_2 - b_2 z_2 - m x_1 y_1 + m b_1 z_1 + u_3 \end{cases} \tag{3}$$

We define the controllers as follows

$$\begin{cases} u_1 = -a_2y_2 + a_2x_2 + ma_1y_1 - ma_1x_1 + mw_1 - k_1e_1 \\ u_2 = -c_2x_2 + a_2x_2 + x_2z_2 - c_2y_2 - mx_1z_1 + mc_1y_1 - k_2e_2 \\ u_3 = -x_2y_2 + b_2z_2 + mx_1y_1 - mb_1z_1 - k_3e_3 \end{cases} \tag{4}$$

Here k_1, k_2 and k_3 are positive constants representing control gain. Hence, by substituting (4) into (3) one can obtain

$$\begin{cases} \dot{e}_1 = -k_1e_1 \\ \dot{e}_1 = -k_2e_2 \\ \dot{e}_1 = -k_3e_3 \end{cases} \tag{5}$$

From system (5), one can easy see that $e_1(t) \to 0$, $e_2(t) \to 0$ and $e_3(t) \to 0$ when $t \to \infty$. This means that the above designed reduced-order projective synchronization scheme can be achieved.

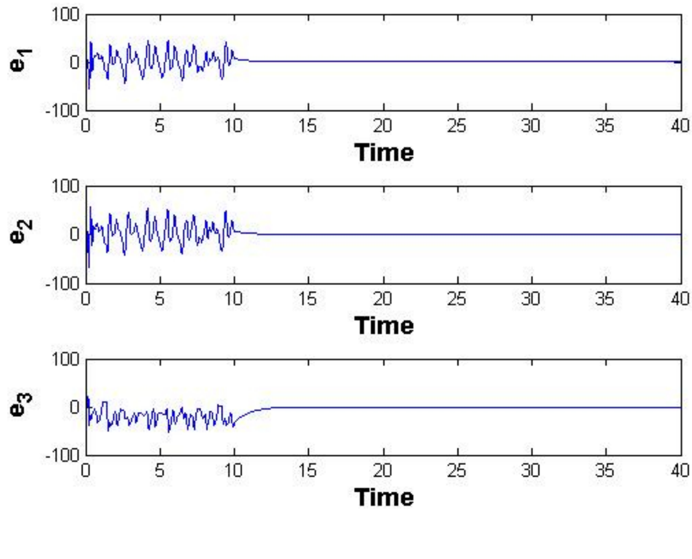

Figure 3
Synchronization Error Signals between Hyper-Chaotic LÜ System and Chen System (The Controller is Activated when $t \geq 10s$.)

3. NUMERICAL SIMULATIONS

In this section, some numerical simulations are presented to illustrate the feasibility and effectiveness of the above designed reduced-order projective synchronization

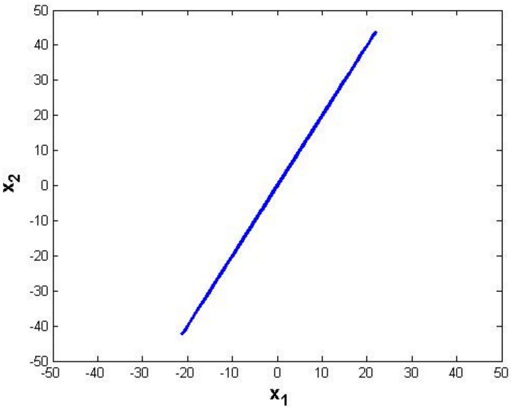

Figure 4
Steady-Phase Diagram of State Variables on (x_1, x_2) Plane

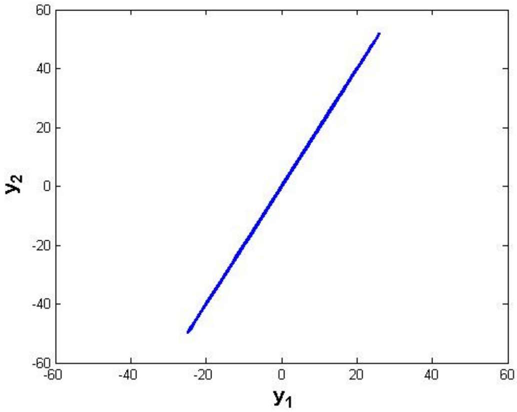

Figure 5
Steady-Phase Diagram of State Variables on (y_1, y_2) Plane

scheme. In the numerical simulation process, the well-known fourth-order Runge-Kutta method is used to solve systems (1) and (2) with controllers (4) and time step size 0.01. The values of parameters are chosen as $a_1 = 36$, $b_1 = 3$, $c_1 = 20$, $r_1 = 1$, $a_2 = 35$, $c_2 = 28$, $b_2 = 3$. The initial conditions and control gain are taken as $x_1(0) = 1$, $y_1(0) = 1$, $z_1(0) = 1$, $w_1(0) = 1$, $x_2(0) = 2$, $y_2(0) = 2$, $z_2(0) = 1$, $k_1 = 1$, $k_2 = 1$, and $k_3 = 1$. Let the scaling factor $m = 2$ and activate the controllers when $t \geq 10s$. Numerical simulation results are shown in Figs. 3-7, respectively. Figure 3 shows that the synchronization errors e_1, e_2 and e_3 quickly approach to zero when the controllers are activated at $t > 10s$. The steady-phase diagram on (x_1, x_2) plane is a straight line with the slope equal to 2. This means that the state variable x_2 and state variable x_1 have been synchronized according to scaling factor $m = 2$. Figure 5 and Figure 6 are similar to Figure 4, which indicate that y_2 and y_1 is also achieved synchronization with scaling factor 2 as well as the state variables z_2 and z_1. Figure 7 indicate that the trajectories of the state variable

x_2 and the state variable x_1 have same wave shape but the amplitude of the state variable x_2 is twice than that of the state variable x_1 at any time t when the above designed synchronization scheme is achieved. The trajectories of other two sets of state variables (y_2 and y_1, z_2 and z_1), which are similar to Figure 7, are omitted for brevity sake.

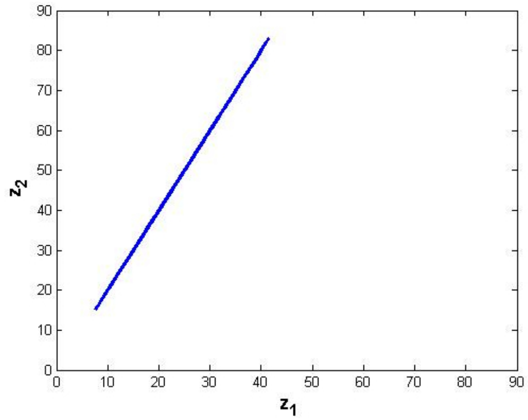

Figure 6
Steady-Phase Diagram of State Variables on (z_1, z_2) Plane

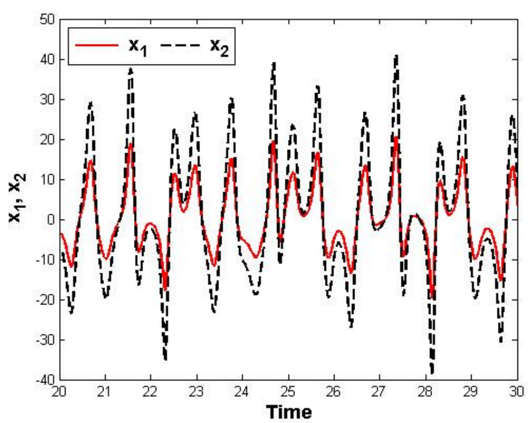

Figure 7
State Trajectories of Variables x_1 and x_2

4. CONCLUSION

In this paper, we designed a reduced-order projective synchronization scheme between fourth-order hyper-chaotic Lü system and third-order chaotic Chen system. A non-zero constant is employed to as the scaling factor. Then we design a nonlinear controller which can successfully control the lower-order response system to synchronize the higher-order drive system according to the given scaling factor. Finally, numerical simulations are carried out to verify the effectiveness and feasibility of the proposed reduced-order synchronization scheme in this paper.

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