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Extension of Some Polynomial Inequalities to the Polar Derivative

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Abstract: Let $p(z)$ be a polynomial of degree n and $D_\propto p(z) = np(z) + (\propto -z)p'(z)$ denote the polar derivative of the polynomial $p(z)$ with respect to the point $\propto$. In this paper we obtain an inequality for the polar derivative of a polynomial which is an improvement of the result recently proved by Mir, Baba and Pukhta (2011) [Thai Journal of Mathematics, 9(2), 291–298].

Key words: Polar derivative; Polynomial; Inequality; Zeros

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1. INTRODUCTION AND STATEMENT OF RESULTS

Let $p(z)$ be a polynomial of degree n, then

$$\max_{|z|=1} |p'(z)| \leq n \max_{|z|=1} |p(z)| \tag{1}$$

Inequality (1) is a well known result of S-Bernstein [2]. Equality holds in (1) if and only if $p(z)$ has all its zeros at the origin. If we restrict ourselves to the class of polynomial not vanishing in $|z| < 1$, then

$$\max_{|z|=1} |p'(z)| \leq \frac{n}{2} \max_{|z|=1} |p(z)| \tag{2}$$

Equality holds in (2) for $p(z) = \propto + \beta z^n$, where $|\propto| = |\beta|$. Inequality (2) was conjectured by Erdös and later verified lax [6].

As an extension of (2) Malik [7] proved that if $p(z) \neq 0$ in $|z| < k$, $k \geq 1$, then

$$\max_{|z|=1} |p'(z)| \leq \frac{n}{1+k} \max_{|z|=1} |p(z)| \tag{3}$$

The result is sharp and equality holds for $p(z) = (z+k)^n$.

Under the same hypothesis, Govil [5] proved that

$$\max_{|z|=1} |p'(z)| \leq \frac{n}{1+k} \left\{ \max_{|z|=1} |p(z)| - \min_{|z|=k} |p(z)| \right\} \tag{4}$$

As a generalization of inequality (3), Dewan and Bidkham [8] proved that if $p(z)$ is a polynomial of degree n having no zeros in $|z| < k$, $k \geq 1$, then for $0 \leq r \leq \rho \leq k$,

$$\max_{|z|=\rho} |p'(z)| \leq n \frac{(\rho+k)^{n-1}}{(k+r)^n} \max_{|z|=r} |p(z)| \tag{5}$$

Dewan and Mir [4] generalized inequality (5) and proved the following result.

Theorem A. If $p(z) = \Sigma_{v=0}^{n} a_v z^v$ is a polynomial of degree n having no zeros in $|z| < k$, $k \geq 1$, then for $0 \leq r \leq \rho \leq k$,

$$\max_{|z|=\rho} |p'(z)| \leq n \frac{(\rho+k)^{n-1}}{(k+r)^n} \left[1 - \frac{k(k-\rho)(n|a_0| - k|a_1|)n}{n(k^2+\rho^2)|a_0| + 2k^2\rho|a_1|} \times (\frac{\rho - r}{k+\rho})(\frac{k+r}{k+\rho})^{n-1} \right] \max_{|z|=r} |p(z)| \tag{6}$$

As an improvement of (6), Aziz and Zargar [3] have obtained the following result.

Theorem B. If $p(z) = \Sigma_{v=0}^{n} a_v z^v$ is a polynomial of degree n, having no zeros in $|z| < k$, $k \geq 1$, then for $0 \leq r \leq \rho \leq k$,

$$\begin{aligned} \max_{|z|=\rho} |p'(z)| \leq & \\ n \frac{(\rho+k)^{n-1}}{(k+r)^n} & \left[1 - \frac{k(k-\rho)(n|a_0| - k|a_1|)n}{(k^2+\rho^2)|a_0| + 2k^2\rho|a_1|} \times \left(\frac{\rho - r}{k+\rho}\right) \left(\frac{k+r}{k+\rho}\right)^{n-1} \right] \max_{|z|=r} |p(z)| \\ -n \left(\frac{r+k}{\rho+k}\right) & \left[\frac{(n|a_0|\rho + k^2|a_1|)}{(k^2+\rho^2)n|a_0| + 2k^2\rho|a_1|} \left\{ \left(\frac{\rho+k}{r+k}\right)^n - 1 - n(\rho - r) \right\} \right] \min_{|z|=k} |p(z)| \end{aligned} \tag{7}$$

The result is best possible and equality holds for the polynomial $p(z) = (z+k)^n$.

As a generalization of inequality (4), Aziz and Shah [10] proved the following.

Theorem C. If $p(z) = a_0 + \Sigma_{v=\mu}^{n} a_v z^v$, $1 \leq \mu \leq n$ is a polynomial of degree n having no zeros in $|z| < k$, $k > 0$, then for $0 \leq r \leq \rho \leq k$,

$$\max_{|z|=\rho} |p'(z)| \leq n\rho^{\mu-1} \frac{(\rho^\mu + k^\mu)^{\frac{n}{\mu}-1}}{(k^\mu + r^\mu)^{\frac{n}{\mu}}} \left\{ \max_{|z|=r} |p(z)| - \min_{|z|=k} |p(z)| \right\} \tag{8}$$

Recently Abdullah, Baba and Pukhta ([9], Theorem 1.1, pp. 293) proved the following more general result.

Theorem D. If $p(z) = a_0 + \Sigma_{v=\mu}^{n} a_v z^v$ in a polynomial of degree n having no zeros in $|z| < k$, $k \geq 1$, then for $0 \leq r \leq \rho \leq k$,

$$\max_{|z|=\rho} |p'(z)| \leq n\rho^{\mu-1} \frac{(\rho^\mu + k^\mu)^{\frac{n}{\mu}-1}}{(k^\mu + r^\mu)^{\frac{n}{\mu}}}$$
$$\left[\left\{1 - \frac{k^\mu(k-\rho)(n|a_0| - k^\mu \mu |a_\mu|)n}{\mu[n|a_0|(\rho^{\mu+1} + k^{\mu+1}) + \mu|a_\mu|(k^{2\mu}\rho + k^{\mu+1}\rho^\mu)]}\right.\right.$$
$$\left.\left(\frac{\rho^\mu - r^\mu}{k^\mu + \rho^\mu}\right)\left(\frac{k^\mu + r^\mu}{k^\mu + \rho^\mu}\right)^{\frac{n}{\mu}-1}\right\} \max_{|z|=r} |p(z)| \tag{9}$$
$$- \frac{(r^\mu + k^\mu)^{\frac{n}{\mu}+1}}{(\rho^\mu + k^\mu)^{\frac{n}{\mu}}} \left\{\frac{(n|a_0|\rho + \mu|a_\mu|k^{\mu+1})}{n|a_0|(\rho^{\mu+1} + k^{\mu+1}) + \mu|a_\mu|(k^{2\mu}\rho + k^{\mu+1}\rho^\mu)}\right.$$
$$\left.\left.\left\{\left((\frac{\rho^\mu + k^\mu}{r^\mu + k^\mu})^{\frac{n}{\mu}} - 1\right) - \frac{n(\rho^\mu - r^\mu)}{\mu(r^\mu + k^\mu)}\right\}\right\} \min_{|z|=k} |p(z)|\right]$$

Equality in (9) holds for the polynomial $p(z) = (z^\mu + k^\mu)^{\frac{n}{\mu}}$, where n is a multiple of μ. Let $D_\propto p(z)$ denotes the polar derivative of the polynomial $p(z)$ of degree n with respect to the point $\propto$, then

$$D_\propto p(z) = np(z) + (\propto - z)p'(z)$$

The polynomial $D_\propto p(z)$ is of degree at most $(n-1)$ and it generalized the ordinary derivative in the sense that

$$\lim_{\alpha\to\infty} \left[\frac{D_\propto p(z)}{\alpha}\right] = p'(z)$$

As an extension of (3) to the polar derivation of a polynomial, we have the following result due to Dewan, Singh and Mir [1].

Theorem E. If $p(z) = a_n z^n + \Sigma_{v=\mu}^{n} a_{n-v} z^{n-v}$, $1 \leq \mu \leq n$, is a polynomial of degree n having no zeros in $|z| < k$, $k \geq 1$, then for every real or complex number α with $|\alpha| \geq 1$,

$$\max_{|z|=1} |D_\propto p(z)| \leq \frac{n}{1+k^\mu} \left\{(|\propto| + k^\mu) \max_{|z|=1} |p(z)| - (|\propto| - 1) \min_{|z|=k} |p(z)|\right\} \tag{10}$$

In this paper we generalize (9) to the polar derivative of a polynomial and which is also an improvement of Theorem E .

Theorem 1.1 If $p(z) = a_n z^n + \Sigma_{v=\mu}^{n} a_{n-v} z^{n-v}$, $1 \leq \mu \leq n$ is a polynomial of degree n having no zeros in $|z| < k$, $k \geq 1$, then for every real or complex number with $|\alpha| \geq \rho$,

$$\max_{|z|=\rho} |D_\propto p(z)| \leq$$

$$n(k^\mu + |\propto|\rho^{\mu-1}) \frac{(r^\mu + k^\mu)^{\frac{n}{\mu}-1}}{(\rho^\mu + k^\mu)^{\frac{n}{\mu}}} \left[1 - \frac{n}{\mu}\frac{A}{B}\left(\frac{\rho^\mu - r^\mu}{\rho^\mu + k^\mu}\right)\left(\frac{k^\mu + r^\mu}{k^\mu + \rho^\mu}\right)^{\frac{n}{\mu}-1}\right] \max_{|z|=r} |p(z)|$$

$$- \left[n(k^\mu + |\propto|\rho^{\mu-1}) \frac{r^\mu + k^\mu}{\rho^\mu + k^\mu}\frac{C}{B}\left\{\frac{\rho^\mu + k^\mu}{r^\mu + k^\mu} - \frac{n}{\mu}\frac{\rho^\mu - r^\mu}{k^\mu + r^\mu} - 1\right\}\right.$$

$$\left. - \frac{n\rho^{\mu-1}}{\rho^\mu + k^\mu}(|\propto| - \rho)\right] \min_{|z|=k} |p(z)| \tag{11}$$

where $A = (k - \rho)(n|a_0| - \mu|a_\mu|k^\mu)k^\mu$

$B = (\rho^{\mu+1} + k^{\mu+1})n|a_0| + \mu a_n k^\mu(k\rho^\mu + k^\mu \rho)$

$C = n|a_0|\rho + \mu|a_\mu|k^{\mu+1}$

Remark 1. For $\rho = r = 1$, inequality (11) reduces to (10)

Remark 2. Dividing the two sides of (11) by $|\propto|$, letting $|\propto| \to \infty$, we get

$$\max_{|z|=\rho} |p'(z)|$$

$$\leq n\rho^{\mu-1} \frac{(r^\mu + k^\mu)^{\frac{n}{\mu}-1}}{(\rho^\mu + k^\mu)^{\frac{n}{\mu}}} \left[1 - \frac{n}{\mu}\frac{A}{B}\left(\frac{\rho^\mu - r^\mu}{\rho^\mu + k^\mu}\right)\left(\frac{k^\mu + r^\mu}{k^\mu + \rho^\mu}\right)^{\frac{n}{\mu}-1}\right] \max_{|z|=r} |p(z)|$$

$$- \left[n\rho^{n-1} \frac{r^\mu + k^\mu}{\rho^\mu + k^\mu}\left\{\frac{C}{B}\left\{\frac{\rho^\mu + k^\mu}{r^\mu + k^\mu} - 1\right\} - \frac{n}{\mu}\frac{\rho^\mu - r^\mu}{k^\mu + r^\mu}\right\} - \frac{n\rho^{\mu-1}}{\rho^\mu + k^\mu}\right] \min_{|z|=k} |p(z)|$$

Equality holds for $p(z) = (z^\mu + k^\mu)^{\frac{n}{\mu}}$, where n is a multiple of μ.

If we take $\mu = 1$ we get the following result which is an improvement of Theorem B.

Corollary 1. If $p(z)$ is a polynomial of degree n and having no zeros in $|z| < k$, $k \geq 1$, $0 \leq r \leq \rho \leq k$, then

$$\max_{|z|=\rho} |p'(z)|$$

$$\leq n\frac{(k + \rho)^{n-1}}{(k + r)^n} \left[1 - n\frac{A'}{B'}\left(\frac{\rho - r}{k + \rho}\right)\left(\frac{k + r}{k + \rho}\right)^{n-1}\right] \max_{|z|=r} |p(z)|$$

$$- \left[n\frac{r + k}{\rho + k}\left\{\frac{C'}{B'}\left\{\frac{\rho + k}{r + k} - 1\right\} - n\frac{\rho - r}{r + k}\right\} - \frac{n}{\rho + k}\right] \min_{|z|=k} |p(z)|$$

where $A' = (k - \rho)(n|a_0| - |a_1|k)k$

$B' = n|a_0|(\rho^2 + k^2) + |a_1|(k^2\rho + k^2\rho)$

$C' = n|a_0|\rho + |a_1|k^2$

Remark 3. If we take $k = 1$ in Theorem 1.1 we get the following result.

Corollary 2. If $p(z) = a_n z^n + \Sigma_{v=\mu}^n a_{n-v} z^{n-v}$, $1 \leq \mu \leq n$ in a polynomial of degree n having no zeros in $|z| < 1$, then for every real or complex number α with

$|\alpha| \geq \rho$,

$$\max_{|z|=\rho} |D_\propto p(z)|$$
$$\leq n(| \propto | + \rho^{\mu-1}) \frac{(1+r^\mu)^{\frac{n}{\mu}-1}}{(1+r^\mu)^{\frac{n}{\mu}}} \left[1 - \frac{n}{\mu} \frac{D}{F} \left(\frac{\rho^\mu - r^\mu}{1+r^\mu} \right) \left(\frac{1+r^\mu}{1+\rho^\mu} \right)^{\frac{n}{\mu}-1} \right] \max_{|z|=r} |p(z)|$$
$$- \left[n \frac{1+|\propto|\rho^{\mu-1}}{1+\rho^\mu} \left\{ \frac{E}{F}(r^\mu+1) \left\{ \left(\frac{\rho^\mu+1}{\rho^\mu+1} \right)^{\frac{n}{\mu}} - 1 \right\} - \frac{n}{\mu} \frac{\rho^\mu - r^\mu}{r^\mu+1} \right\} \right.$$
$$- \frac{n\rho^{\mu-1}}{\rho^{\mu+1}} (|\propto| - \rho) \min_{|z|=1} |p(z)|$$

where $D = (n|a_0| - \mu|a_\mu|)(1-\rho)$
$E = (n|a_0|\rho + \mu|a_\mu|)$
$F = (\rho^{\mu+1}+1)n|a_0| + \mu|a_\mu|(\rho^\mu + \rho)$

2. LEMMAS

For the proof of theorem we need the following lemmas.

Lemma 2.1 Let $p(z) = a_n z^n + \Sigma_{v=\mu}^n a_{n-v} z^{n-v}$, $1 \leq \mu \leq n$ be a polynomial of degree n having no zeros in $|z| < k$, $k \geq 1$, then for every real or complex number α with $|\alpha| \geq 1$

$$\max_{|z|=1} |D_\propto p(z)| \leq \frac{n}{1+k^\mu} \left\{ (|\propto| + k^\mu) \max_{|z|=1} |p(z)| - (|\propto| - 1) \min_{|z|=k} |p(z)| \right\}$$

The above lemma in due to Dewan, Singh and Mir [1].

Lemma 2.2 If $p(z) = a_0 + \Sigma_{v=\mu}^n a_v z^v$, $1 \leq \mu \leq n$ is a polynomial of degree n having no zeros in $|z| < k$, $k > 0$, then for $0 \leq r \leq \rho \leq k$,

$$\max_{|z|=\rho} |p(z)| \leq \left(\frac{k^\mu + \rho^\mu}{k^\mu + r^\mu} \right)^{\frac{n}{\mu}}$$
$$\left[1 - \frac{k^\mu (k-\rho)(n|a_0| - k^\mu \mu |a_\mu|) n}{\mu \left(n|a_0|(\rho^{\mu+1} + k^{\mu+1}) + \mu|a_\mu| + (k^{2\mu}\rho + k^{\mu+1}\rho^\mu) \right)} \right.$$
$$\left. \times \left(\frac{\rho^\mu - r^\mu}{k^\mu + \rho^\mu} \right) \left(\frac{k^\mu + r^\mu}{k^\mu + \rho^\mu} \right)^{\frac{n}{\mu}-1} \right] \max_{|z|=r} |p(z)|$$
$$- \left[\frac{\left(n|a_0|\rho + \mu|a_\mu|k^{\mu+1} \right)(r^\mu + k^\mu)}{n|a_0| \left(k^{\mu+1} + \rho^{\mu+1} \right) + \mu a_\mu \left(k^{2\mu}\rho + k^{\mu+1}\rho^\mu \right)} \right.$$
$$\left. \left\{ \left(\left(\frac{\rho^\mu + k^\mu}{r^\mu + k^\mu} \right)^{\frac{n}{\mu}} - 1 \right) - \frac{n(\rho^\mu - r^\mu)}{\mu(r^\mu + k^\mu)} \right\} \right] \min_{|z|=k} |p(z)|$$

The above lemma is due to Mir, Baba and Pukhta [9].

3. PROOF OF THE THEOREM

Proof. Let $0 \leq r \leq \rho \leq k$, since $p(z)$ has no zeros in $|z| < k$, $k > 0$, then the polynomial $p(\rho z)$ has no zeros in $|z| < \frac{k}{\rho}$, $\frac{k}{\rho} \geq 1$, therefore, by Lemma 2.1 to the polynomial $p(\rho z)$ with $\frac{|\alpha|}{\rho} \geq 1$ gives

$$\max_{|z|=1} |D_{\frac{\alpha}{\rho}} p(\rho z)| \leq \frac{n}{1+(\frac{k}{\rho})^{\mu}} \left[\left\{ \left(\frac{k}{\rho}\right)^{\mu} + \frac{|\alpha|}{\rho} \right\} \max_{|z|=\rho} |p(z)| - \left(\frac{|\alpha|}{\rho} - 1 \right) \min_{|z|=k} |p(z)| \right]$$

i.e.,

$$\max_{|z|=a} |np(\rho z) + \left(\frac{\alpha}{\rho} - z\right) \rho p'(\rho z)| \leq$$
$$\frac{n}{\rho^{\mu}+k^{\mu}} \left(k^{\mu} + |\alpha|\rho^{\mu-1}\right) \max_{|z|=\rho} |p(z)| - \frac{n\rho^{\mu-1}}{\rho^{\mu}+k^{\mu}} (|\alpha| - \rho) \min_{|z|=k} |p(z)|$$

Which is equivalent to

$$\max_{|z|=\rho} |D_{\propto} p(z)| \leq$$
$$\frac{n}{\rho^{\mu}+k^{\mu}} \left(k^{\mu} + |\alpha|\rho^{\mu-1}\right) \max_{|z|=\rho} |p(z)| - \frac{n\rho^{\mu-1}}{\rho^{\mu}+k^{\mu}} (|\alpha| - \rho) \min_{|z|=k} |p(z)|$$

For $0 \leq r \leq \rho \leq k$, we have by using Lemma 2.2,

$$\max_{|z|=\rho} |D_{\propto} p(z)| \leq \frac{n}{\rho^{\mu}+k^{\mu}} \left(k^{\mu} + |\alpha|\rho^{\mu-1}\right) \left(\frac{k^{\mu}+r^{\mu}}{k^{\mu}+\rho^{\mu}}\right)^{\frac{n}{\mu}}$$
$$\left[1 - \frac{n}{\mu}\frac{A}{B} \left(\frac{\rho^{\mu}-r^{\mu}}{k^{\mu}+\rho^{\mu}}\right) \left(\frac{k^{\mu}+r^{\mu}}{k^{\mu}+\rho^{\mu}}\right)^{\frac{n}{\mu}-1} \right] \max_{|z|=r} |p(z)|$$
$$-\frac{n}{\rho^{\mu}+k^{\mu}} \left(k^{\mu} + |\alpha|\rho^{\mu-1}\right)$$
$$\left[\frac{C}{B}(r^{\mu}+k^{\mu}) \left\{ \left(\left(\frac{\rho^{\mu}+k^{\mu}}{r^{\mu}+k^{\mu}}\right)^{\frac{n}{\mu}} - 1 \right) - \frac{n}{\mu} \left(\frac{\rho^{\mu}-r^{\mu}}{r^{\mu}+k^{\mu}}\right) \right\} \right] \min_{|z|=k} |p(z)|$$
$$-\frac{n\rho^{\mu-1}}{\rho^{\mu}+k^{\mu}} (|\alpha| - \rho) \min_{|z|=k} |p(z)|$$

Which is equivalent to

$$\max_{|z|=\rho}|D_{\propto}p(z)| \leq$$

$$n\left(k^{\mu}+|\propto|\rho^{\mu-1}\right)\frac{\left(r^{\mu}+k^{\mu}\right)^{\frac{n}{\mu}-1}}{\left(\rho^{\mu}+k^{\mu}\right)^{\frac{n}{\mu}}}\left[1-\frac{n}{\mu}\frac{A}{B}\left(\frac{\rho^{\mu}-r^{\mu}}{k^{\mu}+\rho^{\mu}}\right)\left(\frac{k^{\mu}+r^{\mu}}{k^{\mu}+\rho^{\mu}}\right)^{\frac{n}{\mu}-1}\right]\max_{|z|=r}|p(z)|$$

$$-\left[n\left(k^{\mu}+|\propto|\rho^{\mu-1}\right)\frac{\left(r^{\mu}+k^{\mu}\right)}{\left(\rho^{\mu}+k^{\mu}\right)}\frac{C}{B}\left\{\frac{\rho^{\mu}+k^{\mu}}{r^{\mu}+k^{\mu}}-\frac{n}{\mu}\left(\frac{\rho^{\mu}-r^{\mu}}{r^{\mu}+k^{\mu}}\right)-1\right\}\right.$$

$$\left.-\frac{n\rho^{\mu-1}}{\rho^{\mu}+k^{\mu}}\left(|\propto|-\rho\right)\right]\min_{|z|=k}|p(z)|$$

This completes the proof of the Theorem. □

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