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Critical Discussion on the Relationship between Failure Occurrence and Severity Using Reliability Functions

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Abstract: In today's competitive world, the basis of many maintenance decisions is their associated risk and this parameter is directly related to occurrence and severity of failures. In this paper, the relationship between the two parameters has been investigated for which, reliability functions has been used. The results imply that this relationship is not compatible with the traditional diagram used in risk analysis approaches such as Failure Modes and Effects Analysis (FMEA) and respectively, decision making based on existing approaches might be risky.
Key words: Diagram; Severity; Occurrence; Risk; FMEA; Reliability function

1. INTRODUCTION

The new view on quality assurance, high expectations of customers and close competition in the international trade, has necessitated manufactures to be more responsible for developing and improving quality. In this regard, one of common approaches is failure mode and effect analysis (FMEA). FMEA is a technique used to identify potential problems “before the event”, and to determine what actions can be taken to prevent them (Shahin, 2004). It is an analytical technique through which all possible “potential failure” modes, the effects that will occur if the failure actually happens and all of the causes which can bring about the failure are determined (Slinger, 1992; Healey, 1994; Slack et al., 2001). It is a group-oriented, structured, and stepwise approach to quantify the effects of possible failures, thus allowing a company to set priorities for action (Vandenbrande, 1998). Main benefits of implementing FMEA are improving the product/process quality and reliability and satisfying customers (Tang and Ho, 1996).

Besides the benefits of FMEA, there are some considerable problems, which have been addressed by many authors and the most important ones could be found in Dale and Shaw (1990), Dale and Cooper (1992), Straker (1995) and Sankar and Prabhu (2001). Problems such as the interdependencies among various failure modes and effects; the assumption that the scales of the three "severity” (S), "occurrence” (O) and "detection” (D) indexes have the same metric and that the same design level corresponds to the same values on different index scales; the assumption that the three indexes are all equally important; and the possibility of identifying situations with the same “risk priority number” (RPN), characterized by different index levels. For example, the condition assigning to (S,O,D) indexes the values (8,1,1) is considered at the

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same level as (2,2,2). Both situations provide an RPN calculation of eight. However, Franceschini and Galetto (2001) debate the legitimacy of this. Although a considerable amount of research has been carried out to enhance the performance of FMEA in the past decade (Xu et al., 2002; Shahin, 2004), most of those methods in the literature have been proposed to enhance the calculation of RPN and it seems that they do not remove the drawbacks of FMEA effectively (Bowles and Pelaez, 1995; Goossens and Cooke, 1997).

One of the major limitations of FMEA and any similar approach, which has not been pointed in any investigation, is that the relationship between two parameters of failure occurrence and severity is considered as negative exponential. In fact, this paper is structured and developed with this argument that not all equipments have the same type of relationship between the two factors.

It is important to note that researchers such as Labib (2004), represented a decision matrix based on the relationship between severity and occurrence. He suggested that the matrix, which he called "decision making grid (DMG)", assist in choosing appropriate maintenance policy. An important message from his investigation is that not all the machinery in a system has similar condition in terms of occurrence (i.e. frequency of failures, which is reverse of mean time between failures) and severity (i.e. mean time to repair) and therefore, different machines require different maintenance policies.

A simple approach to estimate the machinery life probability is to design statistical histograms of machine life (HajShirMohammadi, 2009).The form of distribution functions can be different. The probability of machine life can be calculated through distribution functions of normal, negative exponential and weibal, compatible with the bath tub curve, in which the curve can be divided into three distinct zones or periods quite readily. These zones differ from each other in failure rate and in causation pattern as infant mortality, useful life, and wear-out. The classic bathtub curve against time has three different periods: Decreasing failure rate for infant mortality; Constant failure rate for useful life; and increasing failure rate (without bound) for wear-out. Ebeling (1997) expresses this notion of bathtub curve as a composite of several failure distributions, and formulizes it as ‘a function of piecewise linear and constant failure rates’.

However, as it was mentioned earlier, the main aim of this paper is to show that the relationship between occurrence and severity is not only negative exponential. For this purpose, in the following the traditional relationship between the two factors is demonstrated. Then, reliability functions are introduced and new methodology is developed and discussed.

2. RELATIONSHIP BETWEEN SEVERITY AND OCCURRENCE

Dhillon (1999) studied the relationship between severity and occurrence. He considered only a negative exponential relationship between the parameters. Dhillon suggests criticality index of failure mode for risk analysis. This index was similar to RPN. The relationship between occurrence and severity and the concept of criticality number are depicted in Figure 1.

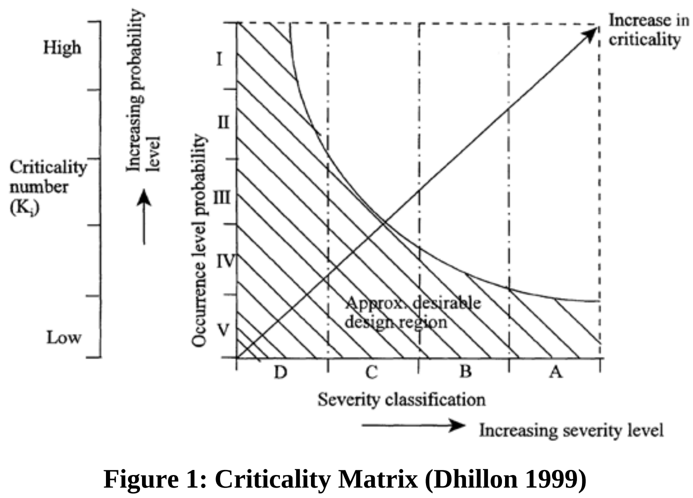

Figure 1: Criticality Matrix (Dhillon 1999)

As it is addressed, as occurrence increases, severity decreases and vice versa. Limbrick (1993), also addresses the same relationship as illustrated in Figure 2. As it is clear, the ISO tolerance lines indicate states of attitude of customers. Towards the end of the severity axis the lines coverage, indicating that a major problem will cause dissatisfaction, probably a customer complaint and closure almost at the same time. The ISO tolerance lines fan out towards the frequency axis to indicate that one can move from one tolerance level or "threshold" to another.

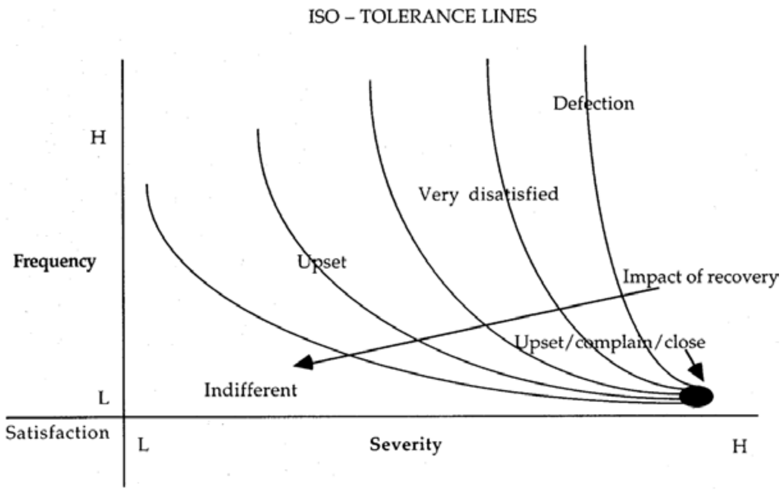

Figure 2: Tolerance of Customers to Service Mistakes (Limbrik, 1993)

Overall Equipment Effectiveness (OEE) is an index which is used for determining the situation of machinery in case of maintenance. It is calculated by multiplying three factors of production rate, availability, and quality rate (Kwon and Lee, 2004). In OEE, the most important factor which is more related to occurrence and severity of failures is availability. Availability states the degree in which process is ready for starting. Operation or machine is not available if it is technically stopped or is being repaired. Considering variety of factors involved in breakdown, different ways could be defined for measuring availability. However, according to literature there exists no function denoting linear relationship between preventive maintenance mode, mode and accessibility, etc. (Lofsten, 2000).

3. RELIABILITY FUNCTIONS

Suppose that T is a non-negative and continuous stochastic variable denoting the useful life of a work piece. Based on probability principles, the life likelihood of product until the maximum time of (t), would be:

$F(t)=P(T\leq t)$

This function is called 'unreliability function'. The subsidiary function is the 'reliability function', which is addressed by R(t) and is the likelihood of the life of the product more than life time of (t):

$R(t)=1-F(t)$

F(t) represents the cumulative life time of product. Derivation of the above function results in probability density function (PDF) of product life time:

$$R(t)=1-F(t) \rightarrow \frac{d(R(t))}{d(t)} = -f(t)$$

Based on lifetime PDF, failure probability at time (t) could be calculated as (Nakhkoob, 2002):

Occurance= $f_{(t)}$

On the other hand, the more failure severity, the more is the time spent on repair and maintenance and the average time of the product progressed between failures will be reduced and in turn, availability will be decreased (Levin and Kalal, 2003).

As it was explained, the bathtub curve includes three time periods in each of which the product lifetime distribution function is different. For various products, it could be approximately argued that the first period has Weibal function with distribution parameter of $\beta<1$; the second period has Weibal function with distribution parameter of $\beta=1$; and the third period has Weibal function with distribution parameter of $\beta>1$ (Zacks, 1992). In the following, various reliability functions are briefly introduced.

3.1 Exponential Distribution

Consider events with constant rate of λ and Poison distribution. Such events are completely random. In this case, the interval of events has exponential function. The reliability function could be defined respectively as:

$$R(t) = e^{-H(t)} = e^{-\lambda t}$$

The mean of time which leads to failure or the mean time between failures is $\frac{1}{\lambda}$ and its standard deviation is $\frac{1}{\lambda^2}$.

3.2 Weibal Distribution

This is one of the most common distributions used in reliability analysis. This distribution is related to some of equipments in which some parts have descending failure rate, some have constant rate and some have increasing rate. This function, which includes almost all of the mentioned modes, is defined as:

$$h(t) = \frac{\beta}{\rho}\left(\frac{t-\gamma}{\rho}\right)^{\beta-1}$$

If $\beta>1$, failures has increasing rate, and if $\beta<1$, they have descending rate. Respectively, the reliability of Weibal distribution can be defined as:

$$R(t) = e^{-\left(\frac{t-\gamma}{\rho}\right)}$$

In the form of $T \approx \mathrm{Wei}(\beta, \gamma, \varphi)$ Weibal distribution, the mean is calculated as:

$$E(T) = MTTF = \gamma + \rho \times \gamma \times \Gamma(1 + \frac{1}{\beta})$$

3.3 Normal Distribution

The probability distribution of a random variable with normal distribution is as follows:

$$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \qquad -\infty < x < +\infty$$

Mean and variance of this distribution are μ, σ^2. Regarding the domain of this distribution, it cannot be used for reliability analysis. Thus a new probability distribution should be computed based on omitting points $(-\infty,0)$. Normal distribution with omitted distance of $(-\infty,0)$ is referred to as truncated distribution. 't' is usually zero and ϕ denotes cumulative redundancy. Consequently, reliability function of the truncated distribution is:

$$R(t)=\frac{1-\phi\left(\frac{t-\mu}{\sigma}\right)}{1-\phi\left(\frac{t_0-\mu}{\sigma}\right)} \qquad t>t_0$$

Generally, it is argued that severe failures which are caused by exhaustion have truncated normal distribution.

3.4 Lognormal Distribution

This distribution can be used in repairing periods of special kind of machinery. The random variable 'X' has lognormal distribution if Y=Ln(X) has normal distribution. This variable is represented as X~Log N(μ) and its density function is calculated as:

$$f_x(x)=\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2} \qquad x<+\infty$$

Mean and variance of lognormal distribution can be calculated similar to the normal distribution. Reliability function of lognormal distribution is computed as:

$$R(t)=\frac{1}{\sqrt{2\pi}}\int_{t_0}^{t}\frac{1}{(t-t_0)}e^{\frac{[\ln(t-t_0)-\mu]^2}{2\sigma^2}}dt$$

4. METHODOLOGY

In manufacturing and maintenance systems, the idle time is critically important and as much as the operating time to failure is higher, as a result availability and effectiveness are higher. In contrast, if the operating time to failure becomes higher, production system will be more affected. In other words, the severity of failure effect becomes higher:

$$\frac{1}{\text{Operating time to failure}}\sim\text{Severity of failure effect}$$

On the other hand, since failure occurrence implies frequency of failure, it can be calculated through the function of equipment life time probability distribution. Reliability means probability of operation of a system more than a specific time, or cumulative probability of equipment operation until specific time. Therefore, failure probability of equipment can be calculated through one minus cumulative probability of equipment operation until specific time. In the following, the above parameters are substituted by severity and occurrence and their interrelationship is estimated based on different reliability functions of the distributions addressed. In order to omit the parameter of time from the two indicators, time is assumed as constant and the functions of lifetime are considered as variable. The diagram of the interrelationships is plotted for each of the functions, using the four distribution functions of exponential, Weibal, normal, and lognormal and considering their corresponding parameters values and machine operating time. The required parameters are calculated and the Matlab software is used for estimation and plotting the diagrams. Sample diagrams of occurrence – severity for the four reliability functions are demonstrated in Figures 3 to 6.

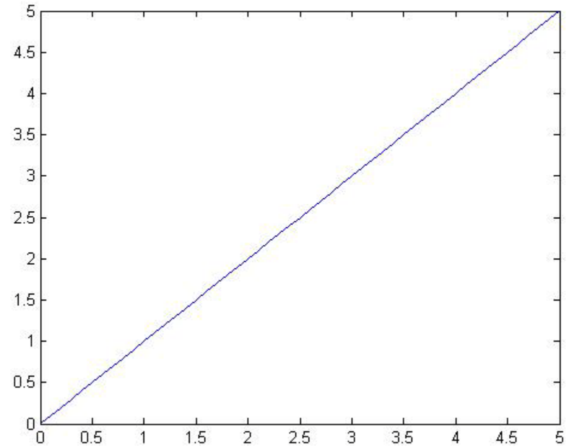

Figure 3: The Diagram of Occurrence – Severity for Exponential Distribution

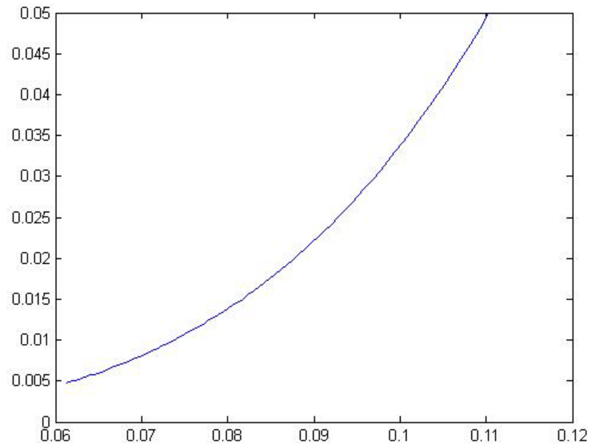

Figure 4: The Diagram of Occurrence – Severity for Weibal Distribution

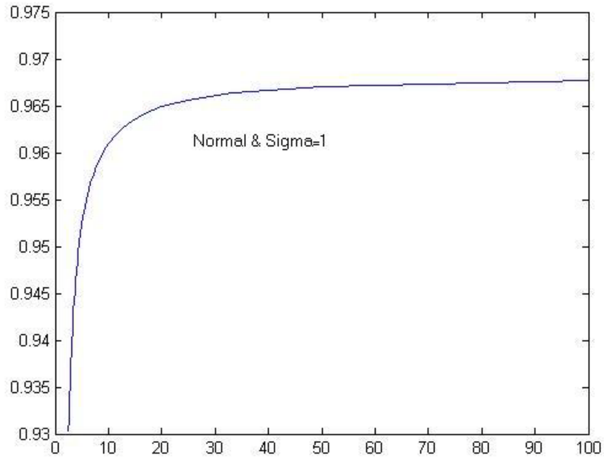

Figure 5: The Diagram of Occurrence – Severity for Normal Distribution

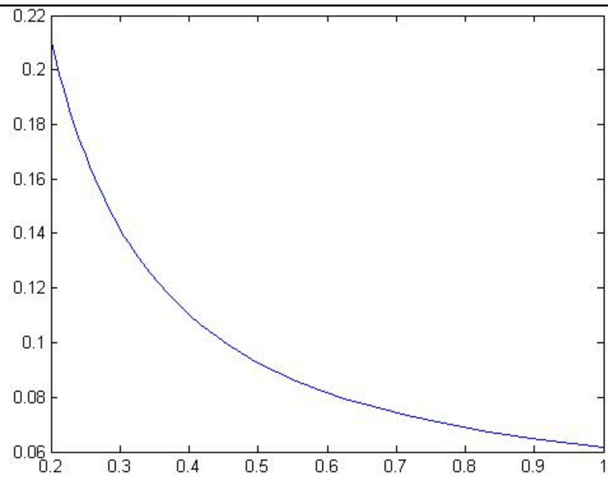

Figure 6: The Diagram of Occurrence – Severity for Lognormal Distribution

5. DISCUSSION

According to the results of this study and particularly in comparing Figure 2 with Figures 3 to 6, it is argued not all existing decision making approaches based on the analysis of the two parameters of occurrence and severity of failures are valid and analyzers need to study the reliability functions prior to design and interpret the diagrams of the interrelationship between the two factors.

It seems the results of this study can provide a new perspective to risk analysis. As it was mentioned in the literature review, one of the problems dealing with techniques such as FMEA is that the relationship between occurrence and severity is not recognized and therefore, equal RPNs might be calculated for different failures, while they are not equal in nature. The results of this study provides a good opportunity to researchers to estimate the interrelationship of severity and occurrence by the use of reliability functions and to revise the traditional approach of considering scale of 1 to 10 for each of the components of RPN, i.e. occurrence, severity and detection rate, which in turn will solve the addressed limitation of equal RPNs.

It is important to note that the proposed methodology does not only evolve FMEA. It can enhance any risk analysis approach of which, parameters of occurrence and severity are at core. For instance, in Hazard and Operability Study (HAZOP) the risk index is calculated simply by multiplying occurrence and severity parameters (similar to FMEA, but detection rate is not included) (Rossing et al., 2010). Whereas, a number of researchers have argued that the HAZOP approach itself has no problem and limitation (Dunjo et al., 2010). However, if the diagram is descending, low amount of the result of multiplied items cannot be indicator of low risk , because it's possible that one parameter be high and another parameter be low (Pillay and Wang, 2003). For instance, if the multiplied value is derived as four, there might be three cases; i.e. two and two, one and four and four and one of the occurrence and severity, respectively. Another example is Decision Making Grid (DMG), in which the equipments/machines are mapped. The performance of the equipment displayed in the grid can be monitored against breakdown and frequency of failures; it also suggests which maintenance policy to apply in order to improve the performance of the equipment/machine (Figure 7). Notice that downtime and frequency can be substituted by mean time to repair (MTTR), and mean time between failures (MTBF), respectively (Labib, 2004).

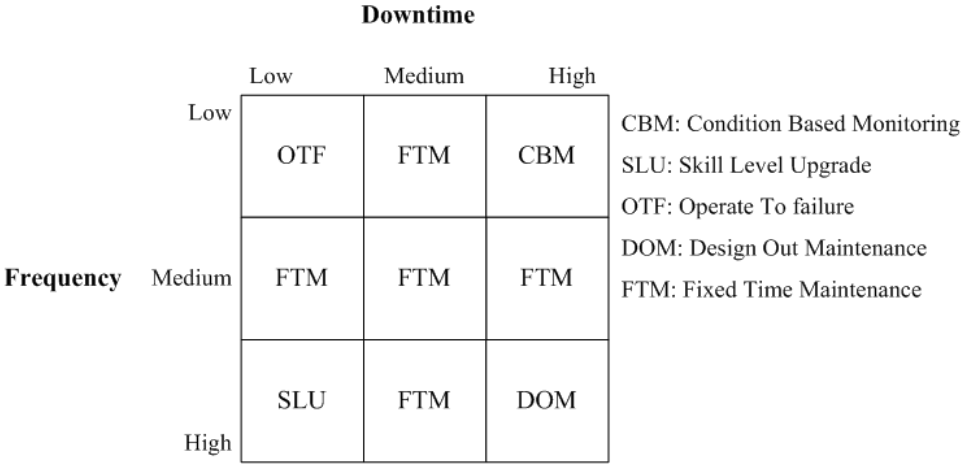

Figure 7: DMG (Fernandez et al., 2003; Labib, 2004)

Comparing DMG with the methodology of this paper and substituting frequency with occurrence and downtime with severity, the four distributions will assist in interpretation of DMG.

For instance, the exponential distribution (Figure 3) denotes two extremes of both low occurrence and severity or both high occurrence and severity, and a linear relationship between the two parameters. This is depicted in Figure 8 and is interpreted that equipments/machines are in their regular life stage and will incrementally subject to failure for which, implementation of fixed time maintenance or Total Productive Maintenance (TPM) is recommended and suddenly they become out of use. Vice versa, when equipments/machines are no longer suitable for run, they are substituted by new ones and the new ones need to be systematically maintained by FTM or TPM and then, once they enter into their regular life, the OTF policy is suggested. Similarly, the sequence of policies depending on the type of the other reliability distributions is represented in Figure 8 and could be interpreted.

Also in Figure 9, the distributions, which are coded in Figure 8 are compared with the bath-tub curve and interesting results are derived. As it is illustrated, the identification of the distribution of failures could play a very important and key role in selection of maintenance policies. For instance, curves 1 and 5 are both used after the equipment/machine is replaced by a new one, but if the failure distribution is computed as lognormal, then only curve-1 should be adopted and there is no guarantee that what would be the next behavior of the failures. In addition, if the failure distribution is computed as normal, then curve-5 should be adopted and it is expected that the system gets into its regular life stage. It is important to note that in this investigation and in Figures 3 to 6, time was assumed as constant, while in Figures 8 and 9, time is variable.

Therefore as it is clear, the new methodology can have a considerable contribution to the evolution and enhancement of risk analysis tools/techniques, policy making approaches, etc. and provides great opportunities to the researchers in future studies. One important topic for future study based on the addressed DMG analysis is to integrate the developed concepts with cost-benefit analysis of maintenance policies as suggested by Labib (1998). In fact, the advantage of the suggested topic over Labib's investigation is that the new methodology will help managers and decision makers in computing and analyzing the cost and benefit of the roadmap of policies (based on the sequences illustrated in Figures 8 and 9) in addition to the cost and benefit analysis of each of the policies, individually.

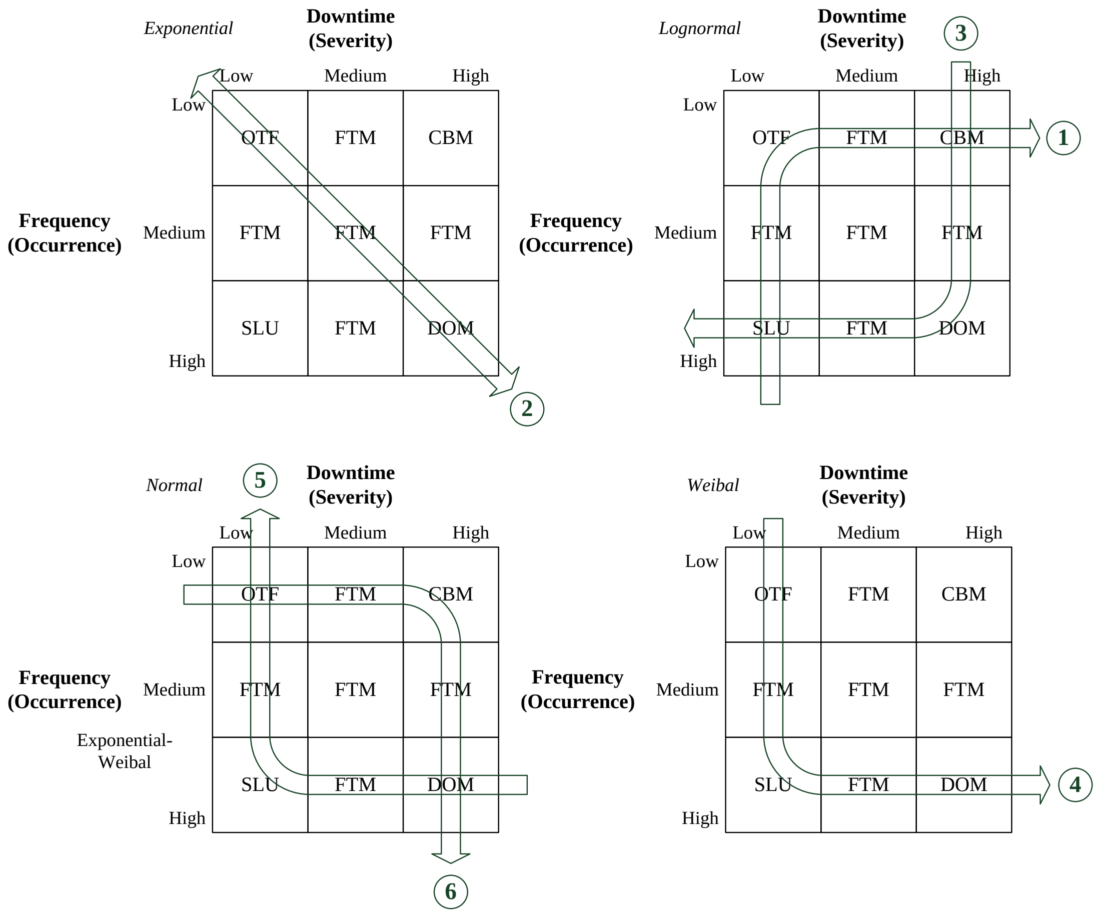

Figure 8: DMG and the Four Related Distributions

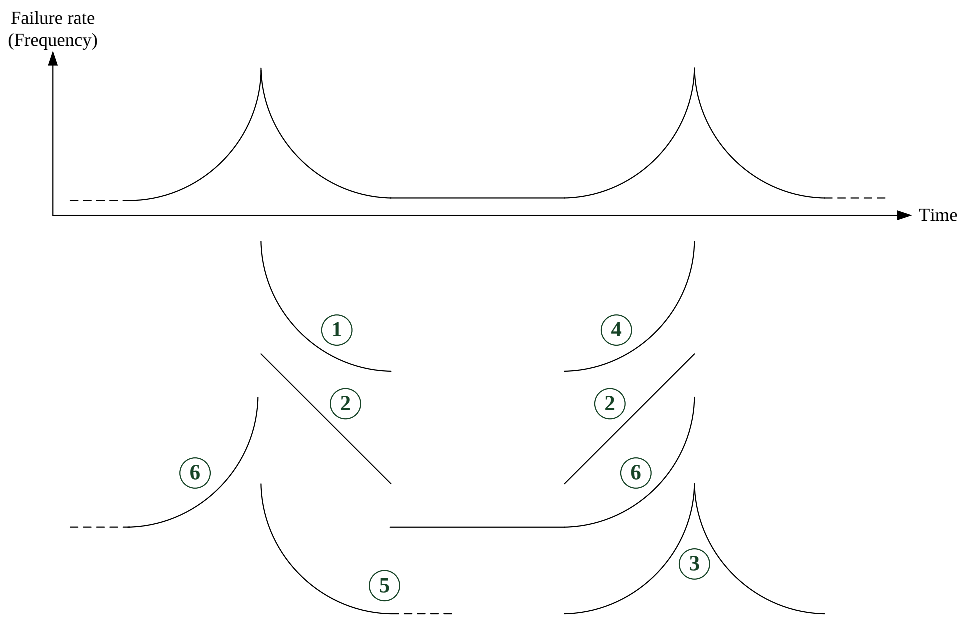

Figure 9: DMG Related Distributions and the Bathtub Curve

6. CONCLUSIONS

In almost all of the existing resources, the interrelationship of frequency and severity is considered as negative exponential, while according to the bath tub curve, this is only related to a particular section of the curve.

In this paper, the relationship between the two parameters of occurrence and severity of failures was investigated for which, reliability functions was used. The results imply that this relationship is not compatible with the traditional diagram used in risk analysis approaches such as Failure Modes and Effects Analysis (FMEA) and HAZOP and consequently, decision making based on existing approaches might be risky. It was also addressed that how the new methodology could evolve and enhance existing tools, techniques and approaches of risk analysis and maintenance policy making such as Decision Making Grid (DMG).

Findings of this research should assist managers in their correct analysis and determination of failure and reliability distribution as well as their roadmap in implementing maintenance policies.

There is enough scope for future work in this research area. In this research, only four well known distributions were studied and it is recommended to researchers to investigate other distributions in order to find if there are any other cases in addition to what was illustrated in Figure 9. The diagrams of the relationship between occurrence and severity were plotted by Matlab software and using conceptual data. However, the methodology needs to be employed in real cases and for this purpose, it is important to have a variety of items/equipments/machines with different failure rate and severity.

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