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Game Analysis of Promoting Substitute Public Goods Provision

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Abstract: A game with parameter for substitute public goods provision is constructed by using non-cooperative game methods. During the voluntary provision of public good, the influence to individual action of the government behaviour and strategic selection of contributors are analysed. Further, incentive mechanism for public goods provision is discussed. For this problem, a cooperative game for public goods provision is constructed, in which the core of game are coincident with Lindahl allocation. So this allocation meets individual rationality and collective rationality.
Key words: Substitute Public Goods; incentive mechanism; core; Lindahl allocation

1. INTRODUCTION

Unlike private goods, public goods are non-competitive and non-exclusive, which tend to have a "free rider" behavior in the course of its private supplies and is not easy to achieve the optimal level of social needs. Olson pointed out first that the voluntary provision of public goods tends to inadequate and leads to the result of "free rider". These call for the implementation of external forces. Hence it is necessary for government (third party)—the main body of implementing external forces, to regulate and incentive the provision of public goods. So many lectures focus on the effective production of public goods by designing the incentive mechanism. Groves and Ledyard (1977) suggested that it can be used "mechanism" process makes the Nash equilibrium solution to achieve efficient Pareto allocation. This mechanism allows players to pursue their own interests and free riding. But mechanism eliminated the incentives. Although such a mechanism can achieve Pareto allocation, it does not meet the individual effectiveness. Hurwicz etc (1980) design a mechanism in the infinite strategy space to solve the individual rationality, which achieve the restrictive Lindahl solution; For this problem, Walker (1981) implement successfully the Lindahl response based on the Nash equilibrium. Lindahl allocation is both individual rationality and collective rationality. Subsequently, there are additional mechanisms have been proposed, such as property privatization, the introduction of tax policy (Felix Bierbrauer, 2009; Liu & Liang 2001). Bagnoli and Lipman (1989) suggest promoting the core mechanism for Discrete and non-exclusive public goods economy. Jackson and Moulin (1992) introduce an efficient and fair mechanism for single indivisible public goods. For multi-public goods, Suresh Mutuswami and Eyal Winter (2004) proposed two sequential mechanisms which meet the efficiency. The payoff obtained in the mechanism is identical to Shapley value of defined cooperative game.

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These Literatures are interest in promoting the production and provision of public goods to avoid "free rider" problem, while ignore the important effect that the government strategies have on promoting rational individuals choose to provide public goods in the implementation process of mechanism.

Substitute public goods means the provision of public goods by participants are indifferencial in the process of their formation. For these goods the provision result is decided by the sum of all provision. A typical example is environmental protection. In this paper the incentive problem and influence of government action on individual action for this kind of public goods is analyzed, in which the government policy is regarded as the disturb parameter (signal). Further the cooperative incentive mechanism for public good provision is discussed.

2. INCENTIVE MODEL FOR PUBLIC GOOD PROVISION

Consider the following N person game G_N . A government (or a social planner) decides to provide a public good y , requiring society's contribution. The total of needed public good is $p(y)$. The government contribution or the policy signal is x , $x \in [\underline{X}, \bar{X}]$. The society is composed by N different individuals indexed by $i = 1, 2, \cdots, N$. Each agent has to decide whether to contribute after observing the policy signal , choosing an indivisible action from the binary set $A_i = \{0 = \textit{not contribnute}, 1 = \textit{contribute}\}$. For convenience, we introduce the following symbols.

Suppose every participant provides only one private good. Let W_i denote the initial property budget of player i and z_i denote the private good provision. According to the budget requirement, it satisfies $z_i + r_i\ p(y) \le W_i$, where $r_i \ge 0$ denotes the public good proportion of player i . Moreover, $\sum_{i \in N} r_i = 1$. Suppose if player don't provide the public good, then all his property will be invested to his private good. Let $\pi_i(1, z_i, x)$ denote the utility of player i when he decides to contribute public good after observing the signal x . On the other hand, if the same player chooses not to contribute (free ride), he will receive utility $\pi_i(0, W_i, x)$. Let $\Delta\pi_i(m, x) = \pi_i(1, z_i, x) - \pi_i(0, W_i, x)$ be the player i 'net payoff from contribution, where m denotes the player number of deciding provide public good in N . So the strategy of every player is

$$\max \quad \Delta\pi_i(m, x) = \pi_i(1, z_i, x) - \pi_i(0, W_i, x)$$

$$\text{s.t.} \quad z_i + r_i\ p(y) \le W_i,\ \sum_{i \in N} r_i = 1,\ r_i \ge 0,\ z_i \ge 0,\ y \ge 0 \qquad (1)$$

In this paper, we regard this public goods supply game participated by government and individuals as a two step game:

First step: the government provides the signal x and individuals decide whether he will provide the public good after observing the government policy signal , namely receiving the private information x_i . Here, for the general, suppose the private information received have certain deviation with actual receipt, namely, $x_i = x + \sigma\varepsilon_i$ (σ is called disturbed factor).

The second step: the players that decide contribute will determine the provision quantity

That is to say, the first step is to consider whether to contribute. So we will analyze the strategy selection of the government and players in game. The second step is essentially to realize the efficient allocation of public goods.

The assumptions about the mechanism are the following:

1) Strategic Substitutes: conditional on the value of x, the greater the other players' strategy profile, the smaller is player i 'incentive to choose the higher action: If $n > n'$, then $\Delta\pi_i(n,x) < \Delta\pi_i(n',x)$ for $\forall x$.

2) Continuity: $\pi_i(1, z_i^M, x)$, $\pi_i(0, W_i, x)$ are continuous function of x and monotonicity, continuous and concave function of z_i .

3) Monotonicity: The greater the value of exogenous variable x, the greater the player i 'incentive to choose the higher action:

if $x, x' \in [\underline{X}, \overline{X}]$, $x \geq x'$, then $\exists c > 0$, s.t. $\Delta\pi_i(n,x) - \Delta\pi_i(n,x') \geq c(x - x')$ for $\forall n$.

4) Upper and Lower Indifference Signals: If other players are choosing identical actions, there exists a unique value of x such that player i is indifferent between the two actions:

$\forall i$, $\exists \underline{k}_i > \underline{X}$, s.t. $\Delta\pi_i(0, \underline{k}_i) = 0$ and $\exists \overline{k}_i$, $\overline{X} > \overline{k}_i > \underline{k}_i$, s.t. $\Delta\pi_i(N-1, \overline{k}_i) = 0$.

5) Player Order: Player j will be "greater" than player i, if for both players observing the same value of x and facing the same strategy profile, player j has less incentive to pick the higher action (i.e. gets a lower net payoff): There exists a player order $\{1, \cdots, N\}$ such that $\exists \alpha > 0$ s.t. if $j > i$ then $\Delta\pi_i(n,x) - \Delta\pi_j(n,x) > \alpha$ for $\forall i, j, n$.

Assumption 1) states the condition in the payoff structure such that this game is a game of strategic substitutes. In general, the greater the other players' strategy profile, the smaller is player's incentive to increase his strategy. Assumption 2) establishes a continuity condition in the government contribution variable (the exogenous parameter), while 3) establishes that the higher the government's contribution, the greater the player's incentive to contribute. Assumption 4) requires that for a sufficiently high (low) values of the government contribution, player i will always (never) contribute, i.e. (not) contributing is a strictly dominant strategy. From assumption 5), we know that if two players face the same strategy profile and the same value of x, the "greater" player will get a lower net payoff.

3. BASIC CONCLUSION

First step: Discuss individual's strategy behavior in public good provision game.

Lemma 1: There exists a unique $\tilde{x} \in [\underline{X}, \overline{X}]$ solving $\Delta\pi_i(n, \tilde{x}) = 0$. Therefore, for $\forall i, \forall n$, $\Delta\pi_i(n,x) < 0$ when $x < \tilde{x}$ and $\Delta\pi_i(n,x) > 0$ when $x > \tilde{x}$.

Proof: Since $\Delta\pi_i(\cdot, x)$ is continuous and monotonic, then $\exists \tilde{x}$ s.t. $\Delta\pi_i(n,\tilde{x}) = 0$ and $\Delta\pi_i(n,x) < 0$ for all $x < \tilde{x}$ and $\Delta\pi_i(n,x) > 0$ for all $x > \tilde{x}$. By assumption 4), we know that $\tilde{x} \in [\underline{X}, \overline{X}]$ for $n = 0$ and for $n = N-1$. Therefore by strategic substitutes 1), $\tilde{x} \in [\underline{X}, \overline{X}]$ for $\forall n$.

From lemma 1 we know that for all n there exists a unique $\tilde{x}$ such that player i is indifferent between the two actions, i.e. given n , player i 's best response is to switch from the lower action to the higher action at a unique value of the signal. Given assumption 3) we can also conclude that the net payoff function is monotonic in x and by assumption 4) we know that for different n the net payoff functions are not intersect each other.

Lemma 2: $\exists \sigma_0 > 0$ s.t. $\forall \sigma \in (0, \sigma_0)$, $\forall i, j, n$, if $j > i$ and $x_j - x_i < \sigma$, then

$\Delta\pi_i(n,x_i)-\Delta\pi_j(n,x_j)>0$.

Proof: From assumption 5) we know that there exists a players order $\{1,\cdots,N\}$ such that $\exists\alpha>0$ s.t. if $j>i$ then $\Delta\pi_i(n,x)-\Delta\pi_j(n,x)>\alpha$ for $\forall i,j,n$. Hence using assumption 3) we know that $\forall j\neq i$ if $x_i<x_j$, then $\exists\sigma'_{ji}>0$ for $\forall n$ s.t. $\Delta\pi_i(n,x_i)-\Delta\pi_j(n,x_i+\sigma'_{ji})=0$. Let $\sigma_0=\min_n\{\sigma'_{ji}\}_{j\neq i}$ therefore $\forall\sigma\in(0,\sigma_0)$ if $j>i$ and $x_j-x_i<\sigma$ then $\Delta\pi_i(n,x_i)-\Delta\pi_j(n,x_j)>0$.

From assumption 5), we know that if two players face the same strategy profile and the same value of x, the "greater" player will get a lower net payoff. This lemma states that this is still true even when they face different values of x, such that their difference is less than σ_0.

According to lemma 1, we give the definition of switching strategy.

Player i takes an action $s_i(x)=a_i\in\{0,1\}$ receiving a signal x_i. A pure strategy profile is denoted as $s=(s_1,s_2,\cdots,s_N)$ where $s_i\in S_i$

Definition 1: A strategy s_i is a switching strategy if $\exists k_i$ s.t. $s_i(x_i)=\begin{cases}1, & if\ \ x_i>k_i\\ 0, & if\ \ x_i<k_i\end{cases}$.

We write $s_i(\cdot;k_i)$ to denote the switching strategy with switching threshold k_i.

Harrison-V, R.J. (2003) conclude that in complete information, for substitute public good provision game each player uses a switching strategy $s_i(\cdot;x_i^*)$ with switching threshold x_i^* which solve the following equation $\Delta\pi_i(i-1,x_i^*)=0$. This paper extends this result to the incomplete information. From lemma 1 we know that for all i, x_i^* not only exists, but it is also unique. Combined with lemma 2, we can get the anomonly result in incomplete information. Let s^* be the profile such that each player is using a switching strategy $s_i(\cdot;x_i^*)$. The result is the following theorem:

Theorem 1: Consider a game G_N satisfying assumptions 1) to 5), then $\exists\sigma_0>0$ s.t. $\forall\sigma\in(0,\sigma_0)$, there exist a unique equilibrium $\{s^*\}$, $s_i(\bullet;x_i^*)$.

This proposition gives the balance result of game, which indicates that each player uses a switching strategy $s_i(\cdot;x_i^*)$ with switching threshold x_i^*.

Second Step: cooperative incentive mechanism for public good provision.

According to lemma 1, we assume that there are m players decide contribute the public good for $\forall x\in[\underline{X},\overline{X}]$ and denote M be the participants set. Thus, we can discuss the rational allocation problem only in set M. So we denote the variable in M by superscript M. From assumption 4), $M=\phi$ (empty set) if $x\le\min_i\{\underline{k}_i\}$. In general, assume $x>\min_i\{\underline{k}_i\}$ and $\Delta\pi_i(m,x)=\pi_i(1,z_i^M,x)-\pi_i(0,W_i,x)>0$.

This time the strategy of each player of provision is:

$$\max\quad \Delta\pi_i(m,x)=\pi_i(1,z_i^M,x)-\pi_i(0,W_i,x)$$

$$\text{s.t.}\quad z_i^M+r_i^M p(y)\le W_i,\ i\in M,\ M\subseteq N,\ \sum_{i\in M}r_i^M=1,\ r_i^M>0,\ z_i^M\ge 0, \qquad (1^*)$$

Zhang Yonglin (1997) states that the core of public good distribution cooperative game is the same with Lindahl allocation in no externalities property under the action of incentive mechanism. This conclusion shows that the core allocation of resource is the same with Lindahl allocation which realizes the individual rationality and collective rationality. This kind of circumstance the social resource achieves Pareto optimality. Thus this allocation is feasible and stable.

Therefore, we construct cooperative game model for public good provision and prove its core is exist. At the same time, we show the core and Linda's configuration is unified, thus is stable.

Define the cooperative game for players in T , $T \subseteq M$:

$$v(T) = \max_{r_1, r_2, \cdots, r_t} \sum_{i \in T} \Delta\pi_i(m, x) = \max_{r_1, r_2, \cdots, r_t} \sum_{i \in T} [\ \pi_i(1, z_i^T, x) - \pi_i(0, W_i, x)] \qquad (2)$$

s.t. $z_i^T + r_i^T p(y) \le W_i$, $i \in T$, $T \subseteq M$

$\sum_{i \in T} r_i^T = 1$, $r_i^T > 0$, $z_i^T \ge 0$

Let $U_i^M(y)$ be the player i 's utility brought from public good y . When given $p(y)$, this utility for every contributor don't change with the number of contributor, so we denote it by $U_i(y)$. According to strategy equivalent principle, the cooperative game defined by (2) is equivalent to the following game

$$u(T) = \max_{r_1, r_2, \cdots, r_t} \sum_{i \in T} [\ U_i(y) - r_i^T p(y)] \qquad (3)$$

Definition 2: The core for m person cooperative game v is imputations satisfying the following condition:

$$C(v) = \left\{ (r_1, r_2, \cdots, r_M) \mid \sum_{i \in M} r_i = v(M), \sum_{i \in T} r_i \ge v(T), \forall T \subset M \right\}$$

Definition 3: A cooperative game is a convex game if it satisfies $v(S \cup \{i\}) - v(S) \ge v(T \cup \{i\}) - v(T)$ for $\forall S, T \subset M$ and $T \subset S \subset M \setminus \{i\}$, $i \in M \setminus S$.

Lemma 3：If a m person cooperative game is convex game , then its core exist.

Lemma 4：Cooperative game (3) is convex game, so its core exists.

Proof: According to the constraints $\sum_{i \in T} r_i^T = 1s$ in (3), the formula (3) can be written:

$$u(T) = \max_{r_1, r_2, \cdots, r_t} \sum_{i \in T} [\ U_i(y)] - p(y)] \ , \ \forall T \subseteq M$$

Then for $\forall S, T \subset M$, $T \subset S \subset M \setminus \{j\}$, $j \in M \setminus S$, we have

$$u(S \cup \{j\}) - u(S) = \max_{r_1, r_2, \cdots, r_{t+j}} \sum_{i \in T \cup \{j\}} U_i(y) - \max_{r_1, r_2, \cdots, r_t} \sum_{i \in T} U_i(y) = U_j^S(y)$$

$U_j^S(y)$ denotes the cooperative net payoff of player j obtained from coalition S . Similarly, we have :

$$u(T \cup \{j\}) - u(T) = U_j^T(y)$$

Because of $r_j^S \le r_j^T$, it can be obtained $z_j^S \ge z_j^T$. The utility function $\pi_j(1, z_j^M, x)$ is increasing function of private good, which shows $\pi_j(1, z_j^S, x) \ge \pi_j(1, z_j^S, x)$.

So $U_j^S(y) \ge U_j^T(y)$. Thus $u(T)$ is convex game and its core exists.

Theorem 2：In substitute public good provision game, the core of cooperative game (3) is coincident with Lindahl allocation of game (1).

AN EXAMPLE

Take a simple example to illustrate results obtained above. Assume a sewage treatment plants will be constructed and the needed funds is 40 million. Assume the government invests in his project and encourages private enterprises and social public to participate. The investment scope of the government is $[0,3]$. For simple we assume there are two individuals in the game and the utility obtained by them are 20 and 30 million respectively. In complete information, the payoff matrix is the following:

Tab. 1: Payoff matrix of invest game

		Individual 2	
		Contribute	Not contribute
Individual 1	contribute	$2-r_1(4-x)$, $3-r_2(4-x)$	$x-2$, 3
	Not contribute	2 , $x-1$	0，0

We can obtain the following conclusions:

1) The switching point of individual 1 is $x_1^* = 2$ and the individual 2's is $x_2^* = 1$. These two points are Indifference.

2) When the investment $x<1$ for government, the selection of two individuals is (Not contribute, Not contribute). When $x>2$, their selection is (contribute, Not contribute).When the investment is in $[1,2]$, the optimal strategy of the two individuals is (Not contribute, contribute).

The results show that the government has great influence on the investment decision-making of individuals. The more the government invests the more incentive the individuals have to provide. But for government, he must master the degree of investment.

If $x \in [1,2]$ and two individuals decide to participate investment, then how to distribute the remaining investment can make the results more stable.

According to theorem 2，the proper allocation can be selected by negotiation. This allocation should be in the core. For the example,

$$v(\{1,2\}) = 2-r_1(4-x) + (3-r_2(4-x)) = 1+x \ , \ v(\{1\}) = x-2 \ , \ v(\{2\}) = x-1$$

So the core of cooperative game is

$$C(v) = \{2-t(4-x),\ 3-(1-t)(4-x) \mid 0<t<1\}$$

If take equal Principle, the allocation is $\{2-\frac{1}{2}(4-x),\ 3-\frac{1}{2}(4-x)\}$.

CONCLUSION

During the process of provision of substitute public goods, the "Chicken Game" and "Pigs' payoffs" game are often appeared. Moreover it is easy to generate non-supply tendency when the participants are the same strength. It is necessary for government to intervene the provision of public goods so as to realizing the Pareto allocation. In the game, the government, as the leading and main body, the key role is to handle the policy signals. The intensity of the policy signals is influence in helping on rational individual to offer public goods. The better the policy, the better the stimulation on rational individual. Undoubtedly, high cost is required. It is also restricted by rent-seeking, inefficient supply, information asymmetry and so on. In this conflict process the government policy should be balanced and stable relatively.

On the other hand, the core of cooperative game is the same with Lindhal allocation of public goods provision game in no externalities property under the action of incentive mechanism. So the Lindhal allocation can be achieved by realizing the core solution of cooperative game. Under this circumstance, the results of game satisfy individual rationality and collective rationality.

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