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Hesitant Fuzzy Multi-Attribute Decision Making Based on TOPSIS With Entropy-Weighted Method

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Abstract
Hesitant fuzzy set (HFS), which allows the membership degree of an element to be a set of several possible values, it has attracted more and more attention due to its powerfulness in representing uncertainty. In this paper, we proposed an approach based on TOPSIS and entropy-weighted method for solving multi-attribute decision making (MADM) problems under hesitant fuzzy environment and the attribute weights is complete unknown. First, we introduce the basic concepts of HFSs. Then, we determine the attribute weights through entropy-weighted method under hesitant fuzzy information. Then, the similarity degree of every alternative with hesitant fuzzy positive ideal solution is displayed to rank all the alternatives. Finally, a numerical example is given to illustrate the effectiveness and feasibility of the proposed method.

Key words: MADM; Score function; HFS; Entropy-weighted method; TOPSIS

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INTRODUTION

Since Zadeh (1965) first proposed the basic model of fuzzy decision making based on the theory of fuzzy mathematics, fuzzy decision making has been receiving more and more attention. Fuzzy set has been successfully applied to handle imperfect and vague information in many different areas. Recently, Torra and Narukawa (2009) and Torra (2010) proposed HFSs as an extension of fuzzy set (Roy, 1996) and analyzed its similarities and differences with intuitionistic fuzzy sets (Atanassov, 1986; Atanassov, 1989; Atanassov, 1994; Wei, 2010; Liu & Wang, 2011), type-2 fuzzy sets (Xia & Xu, 2011; Fazel Zarandi et al., 2012), and fuzzy multisets (Yager, 1986; Miyamoto, 2005; Liu & Wang, 2011). They proposed that permit the membership of an element of a given set having several different values to express uncertain information in the process of decision making, owing to a lack of expertise or insufficient knowledge (Chen et al., 2013). For example, suppose that a company that contains several decision makers is needed to evaluate the quality of an alternative with respect to attribute. Some provide an evaluation of 0.3, some give their evaluation of 0.5, and the other gives an evaluation of 0.9. But the three groups cannot persuade the others to change their opinions, so the evaluation can be represented by the HFS $\{0.3,0.5,0.9\}$.

HFSs have attracted increasingly attention of many scholars in a short period of time because the situations of hesitant are very common in different problems (Rodriguez et al., 2014). Xia and Xu (2011) developed some operators and give their application for solving MADM problem under hesitant fuzzy environment. Wei (2012) proposed prioritized aggregation operators for hesitant fuzzy information, in different priority levels, developed some models for hesitant fuzzy MADM problems. Yu et al. (2012) developed the generalized hesitant fuzzy Bonferroni mean to solve the problems that the attributes are correlative with hesitant fuzzy information. Yu et al. (2013) presented the generalized hesitant fuzzy prioritized weighted average and generalized hesitant fuzzy prioritized weighted geometric operators, then discussed the properties and an example was used to illustrate the effectiveness of the developed method for personnel evaluation. Farhadinia (2013) proposed a score function for ranking HFSs, which meets all the well-known properties of a ranking measure, moreover, and has no counterintuitive examples. Xu and Xia (2011) defined several distance measures and correlation coefficients of hesitant fuzzy sets, and gave the differences and correlations. Xu and Xia (2011) developed the distance and similarity measures for HFSs and proposed an approach based on distance measures for MADM problems.

Hwang and Yoon (1981) first developed TOPSIS to MADM problems. The concept of TOPSIS is that the chosen alternative should have the farthest distance from the negative ideal solution and the shortest distance from the positive ideal solution. The order preference by similarity to TOPSIS is a useful and practical technique for selection of the best alternative and also for the ranking of alternatives (Wang & Elhag, 2006; Boran et al., 2009; Kaya & Kahraman, 2011). Chamodrakas et al. (2011) developed the fuzzy set representation of the closeness to the positive ideal solution and the negative ideal solution and proposed a fuzzy approach based on TOPSIS to rank alternatives in MADM problems. In fact, in the process of decision making, the information about attribute weights always is incompletely known or completely unknown (Xu, 2007). Xu and Zhang (2013) developed an approach based on TOPSIS and maximizing deviation method to deal with the information about attribute weights is incompletely known. However, the existing methods cannot be suitable for dealing with the information about attribute weights is completely unknown, in this paper, we propose an approach to determine the attribute weights under the conditions that the attribute weights are completely unknown, and the attribute values take the form of HFSs.

To do so, the remainder of this paper is organized as follows. In section 2, we review some basic concepts and the corresponding distance measure of HFSs. Section 3 develops the entropy-weighted method and novel TOPSIS for solving MADM problem with hesitant fuzzy information. Section 4 gives the application of the developed approach to see the feasibility of the proposed hesitant fuzzy TOPSIS method. Conclusions are given in the last section.

1. PRELIMINARIES

In the following, we briefly introduce some basic definitions of HFSs.

Definition 1 (Torra & Narukawa, 2009; Torra, 2010) Let X be a fixed set. It is defined as a HFS on X in terms of a function that when applied to X returns a subset of [0, 1].

To be easily understood, Xu and Xia (2011) expressed the HFS by mathematical symbol:

$$E=\left\{\left\langle x,h_E\left(x\right)\right\rangle \middle| x\in X\right\},$$

where $h_E(x)$ is a set of some values in [0, 1], denoting the possible membership degrees of an element $x\in X$ to the set E. And they called $h=h_E(x)$ hesitant fuzzy element (HFE).

It is noted that the number of values in different HFEs may be different, let $l(h_E(x))$ be the number of values in $h_E(x)$

Assumption 1 (Farhadinia, 2013) To avoid any confusion, we should keep in mind the following assumption throughout the article: (1) The arrangement of elements in a HFE h is in an increasing order; (2) For any two HFEs h_1 and h_2, $l(h_1)\neq l(h_2)$. We extend the shorter one by adding the maximum element until both of HFEs have the same length. For instance, let $h_1=\{0.1,0.2,0.3\}$ and $h_2=\{0.4,0.5\}$. Then, we extend h_2 to $h_2=\{0.4,0.5,0.5\}$.

Definition 2 (Farhadinia, 2013) Let $h=\{h_j\}_{j=1}^{l(h)}$ be a HFE, where h_j is in an increasing order and $l(h)$ returns the number of values in Assumption 1. The score function $S(h)$ of a HFE h is defined by

$$S\left(h\right)=\frac{2}{l(h)\left(l(h)+1\right)}\sum_{j=1}^{l(h)} jh_j \in[0,1], \tag{1}$$

Definition 3 Let $h_1=\{h_1^{\sigma(t)}|t=1,2,\cdots l_{h_1}\}$ and $h_2=\{h_2^{\sigma(t)}|t=1,2,\cdots l_{h_2}\}$ be two HFEs and suppose $l=l_{h_1}=l_{h_2}$, then the distance measure between h_1 and h_2 is defined as:

$$d\left(h_1,h_2\right)=\sqrt{\frac{1}{l}\sum_{t=1}^{l}\left(h_1^{\sigma(t)}-h_2^{\sigma(t)}\right)^2}.$$

Definition 4 (Xia & Xu, 2011) Let $H_1=\{h_{11},h_{12},\cdots,h_{1n}\}$ and $H_2=\{h_{21},h_{22},\cdots,h_{2n}\}$ be two HFSs, then we get hesitant weighted Euclidean distance:

$$d\left(H_1,H_2\right)=\sqrt{\sum_{j=1}^{n}\frac{\omega_j}{\sigma_j}\sum_{t=1}^{\sigma_j}\left(h_{1j}^{\sigma_j(t)}-h_{2j}^{\sigma_j(t)}\right)^2},\ \sigma_j=\max(l(h_{1j}),l(h_{2j})) \tag{2}$$

where ω_j is the weight of h_{1j} and h_{2j} and $\sum_{j=1}^{n}\omega_j=1,\omega_j\in[0,1]$.

2. MADM BASED ON TOPSIS AND THE ENTROPY-WEIGHTED METHOD

This section, we will research hesitant fuzzy MADM in which attribute weight is completely unknown, and then a decision-making method which can be applied to evaluate information directly is proposed based on the entropy-weighted method.

2.1 Hesitant Fuzzy Entropy-Weighted Method

Shannon and Weaver (1949) proposed the entropy concept in 1981, which is a measure of uncertainty in information formulated in terms of probability theory. Since the entropy concept is well suited for measuring the relative contrast intensities of criteria to represent the average intrinsic information transmitted to the decision maker (Zeleny, 1982). He proposed the method as:

$$T\left(P_1, P_2, \cdots P_n\right) = -k\sum_{j=1}^{n} P_j \ln P_j, \tag{3}$$

where k is a positive number, usually take k=1, then maximize T(P_1, P_2,$\cdots P_n$).

Because the score of hesitant fuzzy number reflects the degree of ambiguity, so we can calculate the entropy through hesitant fuzzy number score and then obtain the attribute weights. Let $A_i(i=1,2,\cdots,m)$ be a set of m alternatives, $X_j(j=1,2,\cdots,n)$ be the set of n attributes. The hesitant fuzzy decision matrix H can be written as:

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{bmatrix}.$$

Then, by Equation (1), we will get the score matrix:

$$s = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ s_{m1} & s_{m2} & \cdots & s_{mn} \end{bmatrix}.$$

The normalized score matrix denoted as:

$$\hat{s} = \begin{bmatrix} \tilde{s}_{11} & \tilde{s}_{12} & \cdots & \tilde{s}_{1n} \\ \tilde{s}_{21} & \tilde{s}_{22} & \cdots & \tilde{s}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \tilde{s}_{m1} & \tilde{s}_{m2} & \cdots & \tilde{s}_{mn} \end{bmatrix}, \tag{4}$$

where $\tilde{s}_{ij} = \dfrac{s_{ij}}{\sum_{i=1}^{n} s_{ij}}$.

We will obtain the entropy (Wu & Wang, 2014) by:

$$T\left(X_j\right) = -\frac{1}{\ln n}\sum_{i=1}^{n}\tilde{s}\left(h_{ij}\right)\ln\tilde{s}\left(h_{ij}\right). \tag{5}$$

Then the weights of the attribute as:

$$\omega_j = \frac{1 - T\left(X_j\right)}{\sum_{k=1}^{n}\left(1 - T\left(X_j\right)\right)}, j = 1, 2, \cdots, n. \tag{6}$$

2.2 An Approach to Madm With Hesitant Fuzzy Information

Based on hesitant fuzzy sets and the TOPSIS method, an extended TOPSIS method is proposed to rank the preference of alternatives and determined the weights through entropy-weighted. In the process, the hesitant fuzzy positive ideal solution and hesitant fuzzy negative ideal solution are first determined. Then, we will constructed the normalized score matrix to derive the attribute weights. Furthermore, we will calculate the separation measures of each alternatives from the hesitant fuzzy positive ideal solution and the hesitant fuzzy negative ideal solution to obtain the closeness coefficient of each alternative to the positive ideal solution, and then based on the closeness coefficient to rank all the alternatives.

Let $A=\{A_1, A_2, \cdots, A_m\}$ be a finite set of alternatives, where A_i denotes the ith alternative, $X=\{X_1, X_2, \cdots, X_n\}$ be a finite set of attributes, where X_j denotes the jth attribute, the hesitant fuzzy decision matrix $H=(h_{ij})$ for the MADM problems, where h_{ij} represent the judgment of alternative A_i with respect to attribute X_j, then the procedure for hesitant fuzzy TOPSIS method has been given as follows:

Step 1: The hesitant fuzzy positive ideal solution and hesitant fuzzy negative ideal solution (Xu & Zhang, 2013) are defined as follows:

$$A^+=\{h^+_1, h^+_2, \cdots, h^+_n\}, \tag{7}$$

$$A^-=\{h^-_1, h^-_2, \cdots, h^-_n\}, \tag{8}$$

where

$h^+_j=\max\{h^1_{1j}, h^2_{1j}, \cdots, h^{l_{h1j}}_{1j}; \cdots; h^1_{mj}, h^2_{mj}, \cdots h^{l_{hmj}}_{mj}\}$,
$h^-_j=\min\{h^1_{1j}, h^2_{1j}, \cdots, h^{l_{h1j}}_{1j}; \cdots; h^1_{mj}, h^2_{mj}, \cdots h^{l_{hmj}}_{mj}\}$;

Step 2: Utilize Equations (3) and (4), we can obtain the normalized score matrix $\tilde{s} = \left(\tilde{s}_{ij}\right)_{m\times n}$;

Step 3: Utilize Equations (5) and (6); we can calculate the attribute weights ω_j, $j=1,2,\cdots,n$;

Step 4: Utilize Equation (2) to calculate the separation measures $d(A_i, A^+)$ and $d(A_i, A^-)$ of each alternative A_i from hesitant fuzzy positive ideal solution A^+ and the hesitant fuzzy negative ideal solution A^-, respectively.

Step 5: The relative closeness coefficient of an alternative A_i with respect to the hesitant fuzzy positive ideal solution A^+ is defined as the following formula:

$$D\left(A_i\right) = \frac{d\left(A_i, A^-\right)}{d\left(A_i, A^-\right) + d\left(A_i, A^+\right)}. \tag{9}$$

Step 6: Rank the alternatives $A_i(i=1,2,\cdots,m)$ according to the relative closeness coefficients $D(A_i)$, the greater the value $D(A_i)$, the better the alternative A_i.

3. AN ENERGY POLICE SELECTION EXAMPLE

In this section, an energy police selection problem is used to demonstrate the applicability and the implementation process of our approach under hesitant fuzzy environment (Xu & Xia, 2011; Xu & Zhang, 2013; Qian et al., 2013; Kahraman & Kaya, 2010).

Energy is an indispensable factor for the social and economic development of societies. The correct energy policy affects the development of economic and environment, so it is very important to select the most appropriate energy policy. Suppose that there are five alternatives (energy projects) $A_i(i=1,2,3,4,5)$, and four attributes: X_1 technological;X_2: environmental;X_3: socio-political;X_4: economic. Several decision makers are invited to evaluate the performances of the five alternatives. For an alternative under an attribute, although all of the decision makers provide their evaluation values, some of these values may be repeated. However, a value repeated more times does not mean that it has more importance than other values repeated less times. The value repeated one time may be more important than the one repeated twice. To get a more reasonable result, it is better that the decision makers give their evaluations anonymously. Then we only collect all of the possible values for an alternative under an attribute, and each value provided only means that it is a possible value. Then all the possible evaluations for an alternative under the attributes can be considered as a HFE. The results evaluated by the decision makers are contained in a hesitant fuzzy decision matrix, shown in Table 1.

Obviously the numbers of values in different HFEs of HFSs are different. We will extend the shorter one until both of them have the same length. According to the regulations mentioned above, and change the hesitant fuzzy data by adding the maximum numbers as listed in Table 2.

Table 1
Hesitant Fuzzy Decision Matrix

	X_1	X_2	X_3	X_4
A_1	{0.3,0.4, 0.5}	{0.1,0.7,0.8,0.9}	{0.2,0.4,0.5}	{0.3,0.5,0.6,0.9}
A_2	{0.3,0.5}	{0.2,0.5,0.6,0.7,0.9}	{0.1,0.5,0.6,0.8}	{0.3,0.4,0.7}
A_3	{0.6,0.7}	{0.6,0.9}	{0.3,0.5,0.7}	{0.4,0.6}
A_4	{0.3,0.4,0.7,0.8}	{0.2,0.4,0.7}	{0.1,0.8}	{0.6,0.8,0.9}
A_5	{0.1,0.3,0.6,0.7,0.9}	{0.4,0.6,0.6,0.8}	{0.7,0.8,0.9}	{0.3,0.6,0.7,0.9}

Table 2
Hesitant Fuzzy Decision Matrix

	X_1	X_2	X_3	X_4
A_1	{0.3,0.4, 0.5,0.5,0.5}	{0.1,0.7,0.8,0.9,0.9}	{0.2,0.4,0.5,0.5,0.5}	{0.3,0.5,0.6,0.9,0.9}
A_2	{0.3,0.5, 0.5,0.5,0.5}	{0.2,0.5,0.6,0.7,0.9}	{0.1,0.5,0.6,0.8,0.8}	{0.3,0.4,0.7,0.7,0.7}
A_3	{0.6,0.7,0.7,0.7,0.7}	{0.6,0.9,0.9,0.9,0.9}	{0.3,0.5,0.7,0.7,0.7}	{0.4,0.6,0.6,0.6,0.6}
A_4	{0.3,0.4,0.7,0.8,0.8}	{0.2,0.4,0.7,0.7,0.7}	{0.1,0.8,0.8,0.8,0.8}	{0.6,0.8,0.9,0.9,0.9}
A_5	{0.1,0.3,0.6,0.7,0.9}	{0.4,0.6,0.6,0.8,0.8}	{0.7,0.8,0.9,0.9,0.9}	{0.3,0.6,0.7,0.9,0.9}

Step 1: Utilize Equations (7) and (8) to determine the hesitant fuzzy positive ideal solution A^+ and the hesitant fuzzy negative ideal solution A^-, respectively:

A^+={〈0.6,0.7,0.7,0.8,0.9〉,〈0.6,0.9,0.9,0.9,0.9〉,〈0.7,0.8,0.9,0.9,0.9〉,〈0.6,0.8,0.9,0.9,0.9〉},

A^-={〈0.1,0.3,0.5,0.5,0.5〉,〈0.1,0.4,0.6,0.7,0.7〉,〈0.1,0.4,0.5,0.5,0.5〉,〈0.3,0.4,0.6,0.6,0.6〉},

Step 2: Utilize Equations (3) and (4), we will get the normalized score matrix:

$$\tilde{s}=\begin{bmatrix}0.5 & 0.8 & 0.5 & 0.7\\ 0.5 & 0.7 & 0.7 & 0.6\\ 0.7 & 0.9 & 0.6 & 0.6\\ 0.7 & 0.6 & 0.8 & 0.9\\ 0.7 & 0.7 & 0.9 & 0.8\end{bmatrix}$$

Step 3: Utilize the Equations (5) and (6) to obtain the weight vector of attributes:

$$\omega=(0.3,0.1,0.4,0.2)^T.$$

Step 4: Utilize Equation (2) to calculate the separation measures of each alternative from the hesitant fuzzy positive ideal solution and the hesitant fuzzy negative ideal solution, respectively:

$d(A_1, A^+)=0.3$, $d(A_2, A^+)=0.3$,
$d(A_3, A^+)=0.2$, $d(A_4, A^+)=0.2$,
$d(A_5, A^+)=0.2$, $d(A_2, A^-)=0.2$,
$d(A_2, A^-)=0.2$, $d(A_3, A^-)=0.3$,
$d(A_4, A^-)=0.3$, $d(A_5, A^-)=0.3$,

Step 5: Utilize equation (9) to calculate the relative closeness coefficient of an alternative A_i with respect to the hesitant fuzzy positive ideal solution A^+:

$D(A_1)=0.3$, $D(A_2)=0.4$,
$D(A_3)=0.5$, $D(A_4)=0.6$,
$D(A_5)=0.6$.

Step 6: Rank the alternatives $A_i(i=1,2,3,4,5)$ according to the relative closeness coefficients $D_i(i=1,2,3,4,5)$: $A_5 \succ A_4 \succ A_3 \succ A_2 \succ A_1$, thus A_5 is the most desirable alternative.

CONCLUSION

In real-life situations, the judgments or opinions provided by a decision-maker are difficult as an exact numeric number, it usually is fuzzy and uncertainty. It is difficult for decision-maker to give their opinion in the current fuzzy set theory. The HFS is a suitable way to deal with the vagueness of a decision-making's judgments over alternatives with respect to attributes. In this paper, we have first developed a method called entropy-weight method to determine the weights of attributes under hesitant fuzzy environment with the weights of the attribute are completely unknown. An advantage of the proposed method is it deduces the subjectivity influence and remains the original decision information sufficiently. Then we have proposed a new approach based on the TOPSIS to solve MADM problems under hesitant fuzzy environment. The basic concept of the proposed method is based on the hesitant fuzzy positive ideal solution, the hesitant fuzzy negative solution and the relative closeness of each alternative respect to the hesitant fuzzy positive ideal solution to determine the ranking order of all alternatives. Finally, a numerical example for the ranking of alternatives has been discussed to show the effectiveness and feasibility of the proposed method. The approach is has less loss of information and can be applied to managerial decision making problems under hesitant fuzzy environment.

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