Realized Volatility and Stylized Facts of Chinese Treasury Bond Market

LA VOLATILITÉ RÉALISÉE ET LES FAITS STYLISÉS DU MARCHÉ DE BON DU TRÉSOR CHINOIS

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Abstract: Based on high frequency data, this paper studies the volatility stylized facts of Chinese Treasury bond market (CTBM) in detail, including the best sampling frequency selected to compute the realized volatility, the conditional and unconditional distribution of the returns, the long memory property, the intraday, inter-day pattern of the returns and volatility, the asymmetry of volatility, and so on. The main conclusions about CTBM volatility are provided. 15 minute is best sampling frequency. The RV-based conditional distribution of return is nearly normal. Both return and volatility have significant inter-day but insignificant intraday periodicity. Moreover, the volatility asymmetry existing widely in stock or exchange market is not significant in Chinese Treasury bond market.

Key words: Realized volatility, Chinese Treasury bond market, High frequency data

Résumé: Basé sur des données de haute fréquence, le présent article étudie en détail la volatilité des faits stylisés du Marché de bon du Trésor chinois (MBTC), comprenant la meilleure fréquence de prélèvement sélectionnée pour calculer la volatilité réalisée, la distribution conditionnelle et inconditionnelle des retours, la propriété de longue mémoire, le modèle intrajour et interjour des retours et la volatilité, l'asymétrie de volatilité, etc. Les conclusions principales sur la volatilité du MBTC sont les suivantes : 15 minutes est la meilleure fréquence de prélèvement, la distribution conditionnelle RV-basé du retour est presque normale. Le retour et l'asymétrie de volatilité ont tous les deux une périodicité inter-jour signifiante, mais une périodicité intrajour insignifiante. D'ailleurs, l'asymétrie de volatilité existant amplement dans la bourse et le marché des changes n'est pas importante sur le Marché de bon du Trésor chinois.

Mots-Clés: volatilité réalisée, Marché de bon du Trésor chinois, données de haute fréquence

INTRODUCTIONS

In financial time series analysis, it is almost no use to predicate the first moment of the return (or price) of an asset but to study its variance (or volatility). Our ability to estimate time variation in expected returns is hardly improved but we achieve potentially huge gains in our ability to monitor variation in return volatility, or second moments of returns (Andersen et al, 2000). Volatility is an essential ingredient for many applied issues in finance and financial engineering, such as in asset pricing, asset allocation, and risk management (Corsi et al, 2001). After Markowitz quantitative described the volatility firstly in 1952, volatility modeling and forecasting have become one of the most popular topics in finance. Because the volatility of Markowitz is calculated from historical data and is not time-varying, it is also named historical volatility. To describe the time varying of the variance, Engle (1982) proposed the ARCH model. In the following years, the advance of ARCH-based models such as GARCH (Bollerslev, 1986), TARCH (Zakoian, 1994), EGARCH (Nelson,

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1991), GARCH-M (Engle, 1987) etc. give us more capability to solve different special problems in financial markets, such as long-memory of volatility, the effect of conditional variance on conditional mean, or persistence of volatility and so on.

In the last few years, a new method of volatility modeling based on high frequency data was constructed by Andersen, Bollerslev, Diebold, Labys (ABDL hereafter,1997,1999) named realized volatility (RV). Comparing with the former volatility, realized volatility is observable and free of model. ABDL (2000) has shown that by sampling intra-day returns sufficiently frequently, the realized volatility can be arbitrarily closed to the underlying integrated volatility, which is a natural volatility measure. What's more important, based on high frequency data, realized volatility can provide a benchmark to evaluate other volatility models (Andersen et al, 2003).

The study of Chinese capital market based on high frequency data is just the matter of the recent few years with the availability of data. Chinese Treasury bond market (CTBM hereafter) as a market that closely connected to Chinese special economy system is significant to be studied to discover its volatility stylized facts which are different from other markets. CTBM was issued first time in 1981[2]. There was no primary, secondary market and currency before 1987. From 1987 to 1996, OTC transaction was allowed, and mainly on on-exchange transaction. After 1997, the CTBM is mainly on transaction among banks. Now, a market with many kinds of varieties, including both short and long term bond, invertible and un-invertible bond, fixed and flexible yield bond, has shaped gradually. What's more important, the Treasury bond index of Shanghai stock exchange market has been created after Feb.24, 2003. Most of the previous studies of Chinese capital market are focused on the stock market, and little attention has paid to the Treasury market. With the Treasury bond index, deep studies on the market are possible. To discover the volatility stylized facts of CTBM is useful to test the volatility theory and study the investor behaviors in emerging market more deeply as well as to compare it with stock market.

This paper hopes to provide empirical study on the volatility of CTBM based on high frequency data. Modeling the realized volatility of CTBM as well as discovering its typical stylized facts is included. The paper is processed as follows. Section 1 introduces the theory of realized volatility. Then, the best sampling frequency is selected to compute realized volatility of Chinese Treasury bond market in section 2. In section 3, the typical stylized facts of the volatility are discussed in details. Last section concludes.

1. REALIZED VOLATILITY MODELING

1.1 Integrated volatility

As for the volatility estimation (modeling), thought the ARCH volatility or implied volatility were applied widely, the most natural approach is integrated volatility (Andersen et al, 2000). Following the classical hypothesis that logarithmic asset prices follow an univariant diffusion process, letting W be a standard Wiener process and p_k denote the arbitrage-free logarithmic price process (Andersen et al, 2001), then p_k can be written as,

$$dp_k = u_k dt + \sigma dW \tag{1}$$

Or formally,

$$p_k(t) - p_k(t-1) \equiv r_k(t) = \int_{t-1}^{t} u_k(s)ds + \int_{t-1}^{t} \sigma_k(s)dW(s) \tag{2}$$

Now, the standard calculations of quadratic variation, which is unbiased estimator of variance in theory, yield

$$Q\text{var}k(t) = [p_k, p_k]_t - [p_k, p_k]_{t-1} = \int_{t-1}^{t} \sigma^2(s)ds \tag{3}$$

Here, the right part of formula (3), $\int_{t-1}^{t} \sigma^2(s)ds$, is the so-called integrated volatility.

1.2 Realized volatility

Let $P_{t,i}$ denote the j^{th} intra (consider the day-volatility of one security) price of the security in day t and, at sampling frequency Δt, we can construct $N_{\Delta t} = N/\Delta t$ intra-daily returns:

$$R_{t,\Delta t,i} = \ln P_{t,i\Delta t} - \ln P_{t,(i-1)\Delta t}, t = 1,..T, \ i = 1,\ldots,N_{\Delta t} \tag{4}$$

So the daily return is,

$$R_t = \ln P_{t,N_{\Delta t}} - \ln P_{t-1,N_{\Delta t}}, t = 1,\ldots,T \tag{5}$$

Now the realized volatility $v(t_i)$ at time t_i is defined as (Daconogna et al, 2001),

[2] Except the very short time issue immediate after 1949 when the new government of China constructed

$$v(t_i) = \left[\frac{1}{N_{\Delta t}} \sum_{i=1}^{N_{\Delta t}} \left| R_{\Delta t,t,i} \right|^p \right]^{\frac{1}{p}} \qquad (6)$$

The exponent p in formula (6) is often set to 2 so that $v(t_i)^2$ is the variance series of the return with zero drift. When $p=1$, the volatility is just the fine volatility (Müller et al, 1997). ABDL(2000) showed that by sampling intra-day returns sufficiently frequently, the realized volatility could be arbitrary closed to the underlying integrated volatility.

An important hypothesis of formula (6) is that the returns have an expectation significant to be zero. If such an assumption is not satisfied, an alternative of realized volatility definition is as,

$$v(t_i) = \left[\frac{1}{N_{\Delta t} - 1} \sum_{i=1}^{N_{\Delta t}} \left| R_{t,\Delta t,i} - \frac{1}{N_{\Delta t}} \left[\sum_{k=1}^{N_{\Delta t}} R_{t,\Delta t,k} \right] \right|^p \right]^{\frac{1}{p}} \qquad (7)$$

$v(t_i)$ of formula (6) and (7) is the volatility of regularly spaced Δt returns, but what is more important and often used is the scaled form (Daconogna et al, 2001), such as the one-day-volatility or one-year-volatility, which can be calculated as:

$$v_{scaled} = \sqrt{\frac{\Delta t_{scale}}{\Delta t}} . v(t_i) \qquad (8)$$

Here, $v(t_i)$ is computed from formula (6) or (7), and Δt_{scale} is the scaled term (time), such as one day or one year et al.

1.3 Sampling frequency of realized volatility modeling

The theory that realized volatility is the consistent estimation of integrated volatility is appealing, but, unfortunately, the empirical data are not so well. The assumption that log asset prices conform to a diffusion process becomes less realistic as the time scale reduces.

Now considering formula (6) or (7), to compute the realized volatility, the sampling frequency Δt must be selected firstly. On one hand, as the analysis of Andersen et al（1997，2000）, higher frequency can reduce the statistic stochastic error. On the other hand, the microstructure effect produced by bid-ask bounce will make the volatility estimated by formula (6) or (7) be biased and the assumption that the price is a diffusion process will become less realistic when the sampling frequency increased (Corsi, 2001). The following section will show that only if the intra-period returns are serially uncorrelated and the microstructure effect is little, will the realized volatility measures be an unbiased estimator of the average true volatility over the interval of interest. That is to say, the best sampling frequency should be selected at highest frequency with little microstructure effect.

Now supposing that the asset's (excess) return at the daily frequency can be characterized as:

$$R_t = \sigma_t \varepsilon_t \qquad (9)$$

Here, $\varepsilon_t \sim iid\ N(0,1)$ and σ_t^2 represents the day-t return variance. So,

$$E\left[R_t^2\right] = \sigma_t^2 \ ; \ V\left[R_t^2\right] = 2\sigma_t^4$$

Now consider the intra-daily data at sampling frequency Δt, each return of quote or tick, if uncorrelated, can be characterized as:

$$R_{t,\Delta t,i} = \sigma_t \varepsilon_{t,\Delta t,i} \qquad (10)$$

Here, deduced by formula (9) and (10), it well be $\varepsilon_{t,\Delta t,i} \sim iid\ N(0, N_{\Delta t}^{-1})$ and

$$R_t = \sum_{i=1}^{N_{\Delta t}} R_{t,\Delta t,i} \ ; \ E\left[R_{t,\Delta t,i}^2\right] = N_{\Delta t}^{-1} \sigma_t^2$$

The decomposition of the daily return into the sum of $N_{\Delta t}$ intra-daily returns can be used to derive the following equation:

$$R_t^2 = \left[\sum_{i=1}^{N_{\Delta t}} R_{t,\Delta t,i} \right]^2 = \sum_{i=1}^{N_{\Delta t}} R_{t,\Delta t,i}^2 + 2 \sum_{i=1}^{N_{\Delta t}-1} \sum_{j=i+1}^{N_{\Delta t}} R_{t,\Delta t,i} R_{t,\Delta t,j} \qquad (11)$$

When the assumption of uncorrelated returns at sampling frequency Δt is satisfied, the expectation of the second term on right side of formula (11) is zero. Then,

$$E\left[\sum_{i=1}^{N_{\Delta t}} R_{t,\Delta t,i}^2 \right] = \sum_{i=1}^{N_{\Delta t}} E\left[R_{t,\Delta t,i}^2\right] = E\left[\sum_{i=1}^{N_{\Delta t}} R_{t,\Delta t,i} \right]^2 = E\left[R_t^2\right] = \sigma_t^2 \qquad (12)$$

That is, the realized volatility measure will therefore yield an unbiased estimator of the return variance while the intraday returns have no series correlation.

To select the best sampling frequency, Andersen et al (1997) propose a statistics VR as,

$$VR = \frac{N_{\Delta t}.Var\ (R_{t,\Delta t,i})}{Var\ (\sum_{i=1}^{N_{\Delta t}} R_{t,\Delta t,i})} \tag{13}$$

The principal of the statistics VR is simple, if Δt is a proper sampling frequency, the volatility modeled on proper sampling frequency should have the scaling law as demonstrated in Eq.(8). The optimal Δt will produce a VR statistic nearly to one.

2. REALIZED VOLATILITY OF CTBM

Now, the bond index of Chinese Treasury bond market (CTBM) is considered. Tick-by-tick data of Chinese Treasury bond index from Jun.1 2004 to Dec.31 2005 is taken to be analyzed. The data are supported by CSMAR database. There are 391 transaction days with 634489 ticks, about 1622 ticks per day[3]. Here, the same as Andersen et al (2003), we delete the ticks out of transaction time directly. There are three interpolation schemes to be used generally, the previous-tick interpolation, the posterior-tick interpolation and linear interpolation scheme (Daconogna et al, 2001). Because there is no short-sell system in CTBM, no transaction may means bad news (Diamond and Verrecchia, 1987), so the linear interpolation scheme is selected in this paper. Through the linear interpolation method, 95499 1-minute logarithm returns[4] are included as shown in figure.1.

To choose the optimal sampling frequency Δt, with the help of Matlab6.5 software, it is found that the VR statistics is almost close to one (0.9974) with sampling frequency of 15 minutes, so 15-min is selected as the optimal sampling frequency to calculate realized volatility.

With the optimal sampling frequency, the realized volatility of CTBM is computed by equation (6) with Δt equal to 15 minutes, and the daily-scaled volatility gotten from equation (7) is shown in figure.2. The maximal one-day-volatility is about 0.0061 and the minimum is about 2.135×10^{-4}.

[3] As the developing of electronic trading system of Shanghai stock market, more than 16000 ticks of trading can be processed each second, with which the tick-by-tick index data is produced. More information is included in the official web of Shanghai stock market: http://www.sse.com.cn/sseportal/ps/zhs/sjs/jysjs.shtml

[4] All returns in this paper are logarithm ones.

3. THE STYLIZED FACTS OF CTBM VOLATILITY

To describe the stylized facts of volatility is prerequisite to take risk management, volatility forecasting or asset pricing. Relative more studies are focusing on stock or exchange markets. So the questions that whether the volatility stylized facts of CTBM is different from that of other capital markets, or whether the volatility characters existing in stock and exchange market also exist in CTBM, are necessary to be answered. The following section will show the stylized facts of CTBM and compare them with other markets mainly on six aspects, the distribution of realized volatility, the unconditional and unconditional distribution of the return, the long memory character of the realized volatility, the intraday and inter-day pattern of the return and volatility, and the asymmetry of the volatility. After the analysis of the above six analyses, it is found that the volatility of CTBM is different from that of other markets as shown in Andersen et al (1997,1999,2003), Corsi(2001), Daconogna(2001) et al.

3.1 The distribution of realized volatility

Literatures (Andersen et al, 2001, 2003) have found that, the distribution of realized volatility, for foreign exchange market, is right skewed and not normality but the logarithm realized volatility is nearly normal. How about is it in CTBM? As shown in Figure 3, for CTBM, the kurtosis of the logarithm realized volatility is lower than 3, but it is right skewed. The normality can be rejected at 1% significance levels.

It is different from other market with high kurtosis. The relative lower kurtosis of logarithm realized volatility also means that CTBM is rather a calmly market with less suddenly fall or rise of the price.

3.2 The unconditional and conditional distribution of the returns

As mentioned above, supposing the return series are decomposed as $R_t = \sigma_t \varepsilon_t$, where, $\varepsilon_t \sim iid\ N(0.1)$, and σ_t is the time-*t* conditional standard deviation. The σ-standardized return is:

$$\varepsilon_t = \frac{R_t}{\sigma_t} \tag{14}$$

Based on the above definition, the distribution of return standardized by σ_t is so-called conditional distribution. Here, σ_t is unknown and must be estimated. In the past, σ_t can be estimated by history volatility or ARCH (GARCH) models. Here, realized volatility is used.

A relative consistent view of literature about the unconditional distribution of return series is that daily or longer time sampling frequency return series is logarithm normality. And increasing the length of the sampling frequency, to weekly, fortnightly, or monthly, will lead to the reduction of persistence of the conditional variance and kurtosis (Baillie and Bollerslev, 1989). But for intraday return, it is not the case generally. For example, ABDL(2000) found that the unconditional distributions of exchange rate returns are symmetric but highly leptokurtic. Only the conditional covariance $P(RV)$-standardized (multivariate standardization by realized volatility) returns have been eliminated well (Andersen, 2001). That is, $P(RV)$ can capture the main character of exchange rate volatility, but the conditional variance method of ARCH or GARCH cannot do it. As for CTBM, the unconditional distribution of return and standardized return are shown in Table 1.

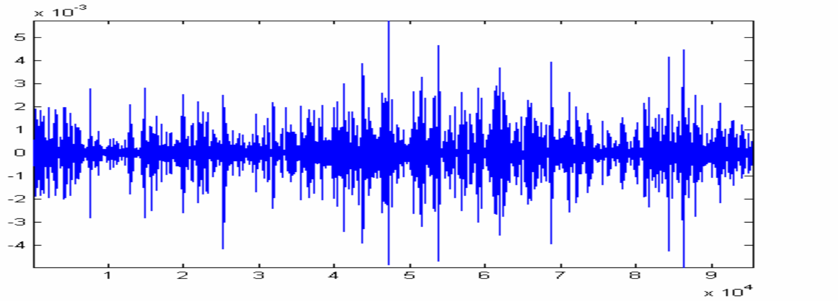

Figure.1 1-min return of Chinese Treasury bond index series

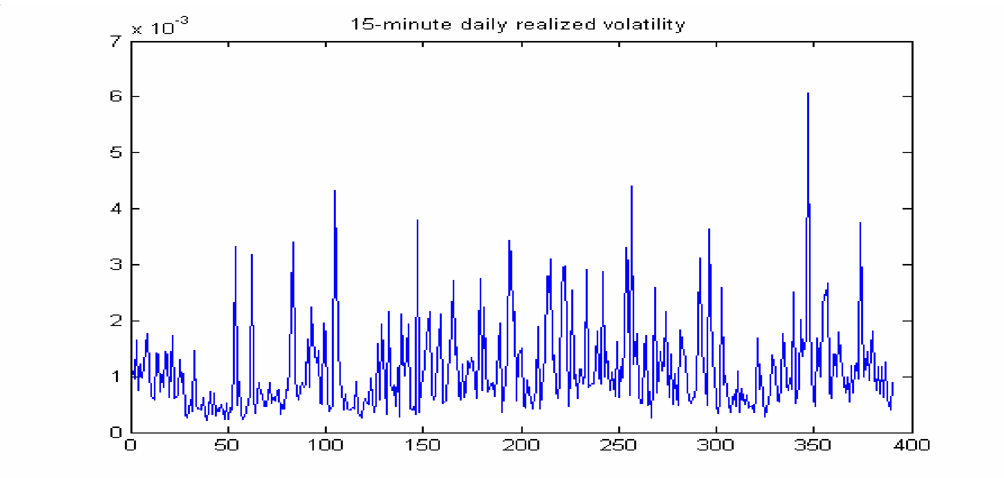

Figure 2 daily volatility of CTBM index with 15-minute sample frequency [5]

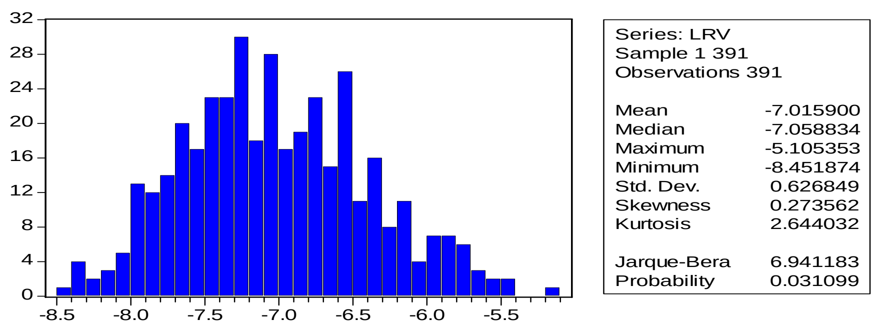

Figure.3 The distribution of logarithm-realized volatility

[5] In the following sections of this paper, without special explanation, "daily volatility" denotes the daily realized volatility computed at sampling frequency of 15 minutes.

Table.1 The unconditional and conditional distribution statistics of daily return

Statistics	Unconditional	$R_t/GARCH(\sigma_t)$	$R_t/EGARCH(\sigma_t)$	$R_t/TGARCH(\sigma_t)$	R_t/RV_t
Mean	0.000291	0.236219	0.245331	0.242669	0.265334
Median	0.000267	0.219103	0.211381	0.218529	0.400543
Maximum	0.005722	3.698168	4.316742	3.849774	3.198687
Minimum	-0.005078	-3.166172	-3.328858	-3.158312	-3.110353
Std. Dev	0.001330	1.001257	1.00273	1.002494	1.144757
Skewness	-0.176394	-0.03849	0.054739	0.01204	-0.210063
Kurtosis	4.833237	3.926884	4.098455	3.910525	2.696738
Jaque-Bera	56.77999	14.095295	19.85248	13.51614	4.373894
Probability	0.000000	0.00087	0.000049	0.001161	0.112259

It can be found that the unconditional distribution of CTBM return series is absolutely different from normality. The conditional distribution of returns standardized by GARCH、EGARCH and TGARCH also have a kurtosis higher than three, the Jaque-Bera statistics and the p-value demonstrate that these conditional distributions are not normal. However, the conditional distribution of returns standardized by realized volatility is nearly normal, as shown in last column of Table 1. The normality of conditional distribution of CTBM return standardized by realized volatility cannot be rejected at even 10% significant levels, which means the realized volatility is a proper way to describe the variation of the return process. To compare the four kinds of conditional distribution with normality, the QQ normality testing graphs is shown in Figure 4.

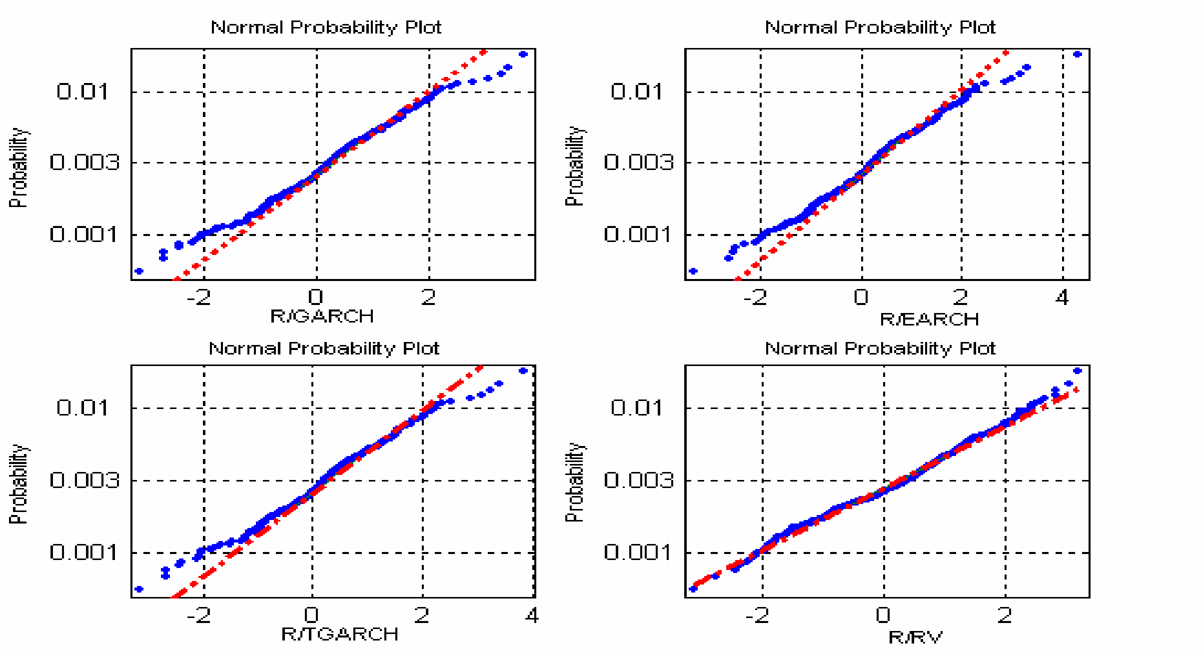

Figure.4 graphical normality testing graphs of conditional distribution of returns

The red dotted line connects the 25th and 75th percentiles of the data. The blue dotted lines describe the percentiles of the sample. If the data conform to normal distribution, the blue line will appear linear and the assumption of normality is reasonable. But, if the data is non-normal, the blue line may follow a curve. It is obvious that the two lines of conditional return distribution standardized by realized volatility are superposed best. That is to say, realized volatility can capture the volatility dynamics of CTBM better than other GARCH, TGARCH or EGARCH model, especial for the character of the tail of distribution. This result is similar, to some extent, to that of ABDL(2000), in which the foreign exchange market is concentrated.

3.3 The long-memory of realized volatility

Volatility of finance time series is usually long-memory (Daconogna et al, 2001). That is, the influence of the market volatility will continue for a long time, which will lead to slowly decline of volatility autocorrelation. The result of Andersen et al (2001) demonstrates that the volatility of the foreign exchange market may be persistent more than 60 transaction days. Long-memory character is a direct reflection of the volatility persistence, which is exiting in financial time series widely. Figure.5 shows the autocorrelation of the daily realized volatility and return of CTBM. It can be found that both the autocorrelation of daily return and volatility presents negative-exponent decline. The first order of autocorrelations of return and volatility are about 0.31233 and 0.25444 respectively, but come to the insignificant bounds at the second or third lag term. The autocorrelations of return and volatility means that, in commonly, the effect of one event can only maintain two days, and the long-memory character is not so obvious compared with that of exchange or stock market.

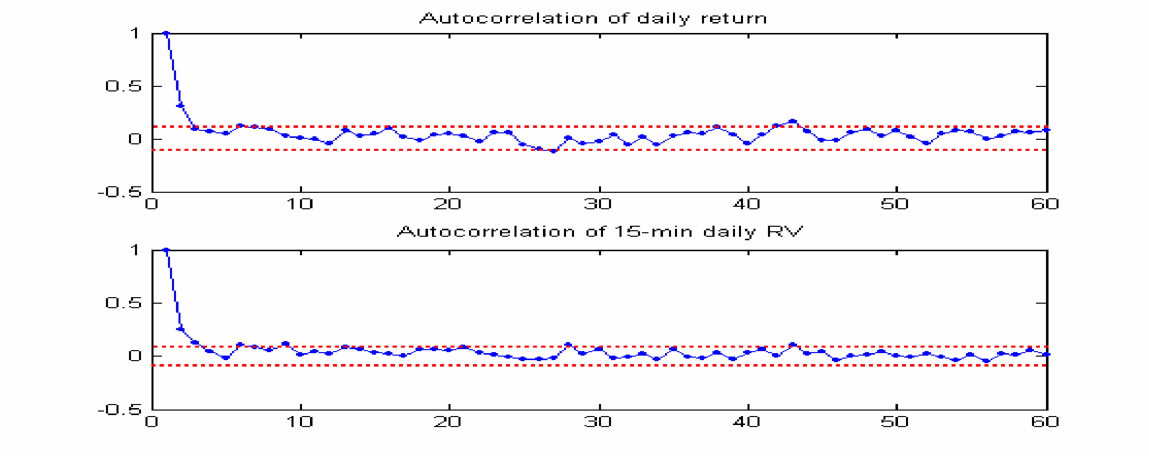

Figure.5 The autocorrelation of daily return and daily RV

Moreover, the short-memory character of CTBM implies that ARMA, ARCH-family models can be used well to model its volatility.

3.4 The intraday and inter-day pattern of returns

Intraday and inter-day periodic pattern widely exist in capital market, which is consistent with market microstructure theories that emphasize on the role of private and asymmetric information in the price formation process. The regular release of macroeconomic news, for example, is one of the reasons that leads to intraday periodic pattern of volatility (Ederington and Lee, 1993).

With the sampling frequency of 15 minutes, a transaction day of 4 hours is divided into 16 sub sections. Figure 6 shows the average 15-minute return of all the 391days in each section. The highest average 15-min return, 6.87×10^{-5}, takes place in the 15 minutes before the break of the noontime, and the lowest one, -4.5×10^{-5}, takes place in the 15 minutes after the re-open of the market in the afternoon.

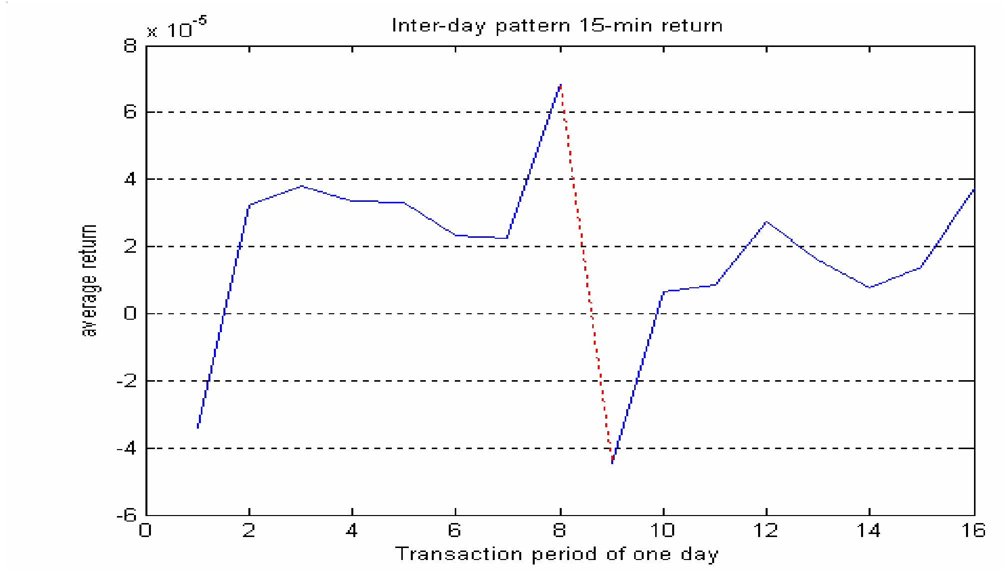

Figure.6 The inter-day pattern of 15-minute return

It can be found that, as for the break of transaction in the noontime, the inter-daily pattern of 15-minute return can be divided into two parts, the morning part (from 9.30 AM to 11:30 AM) and the afternoon part (from 13:00 PM to 15:00 PM). It is interesting that the two parts have almost the same shape that 15-minute return is negative near each opening time and go upward in the last 15 minutes. Moreover, return in the morning time is higher than that in the afternoon, so the afternoon part of the graph is just as a southeast unite-level-move of the morning part. In a word, the intraday pattern of CTBM return is just as double inverse- "Z".

In capital market, it is not a new finding that there is "January effect" (return is higher is January or the few days of a new year.) or "weekly effect" (return has a significant difference in different day of a week.). As to test the weekly periodic pattern in CTBM, average daily return from the transaction day from Monday to Friday in all of the 391 sample section is shown in figure.7.

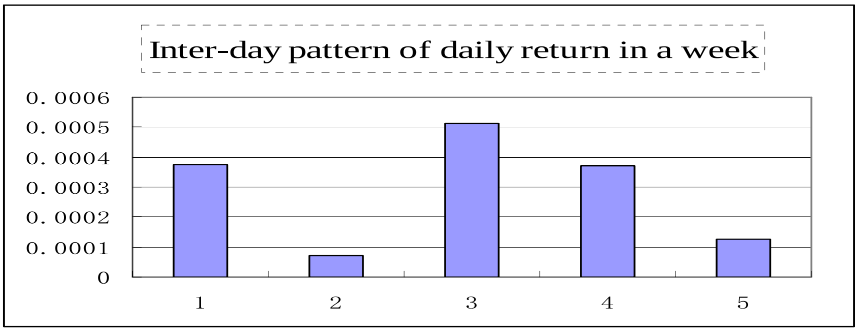

Figure.7 Average daily return from Monday to Friday

Firstly, from figure.9, it can be found that average return in all day of one week is positive. Considering the weekly pattern of return, it can be found that daily return is highest in Wednesday and lowest in Tuesday and Friday, and the highest daily average return of Wednesday is almost five times more than that of Tuesday. To test the significance of weekly periodicity of volatility, the t-statistics of difference of two days is constructed as,

$$t_{i,j} = \frac{\overline{RV_i} - \overline{RV_j}}{Sec(RV_{i,t} - RV_{j,t})} \tag{15}$$

Here, $\overline{RV_i} = \sum_{t-1}^{T} RV_{i,t}$, $RV_{i,t}$ is the day- i return in week t , T is the number of weeks. And the test result is shown in table.2.

Table. 2 Significance test of inter-day pattern of return

Day of week	Mon.	Tue.	Wed.	Thu.	Fri.
Mon.	0				
Tue.	-0.23848	0			
Wed.	-0.1768	0.072755	0		
Thu.	-0.027552	0.20653	0.18258	0	
Fri.	-0.15846	0.071448	0.013717	-0.14139	0

The significance test result of inter-day pattern in Table 1. indicates that, though the variation of daily return, the inter-day pattern is insignificant.

3.5 The intraday and inter-day pattern of realized volatility

Apart from the return, intraday and inter-day pattern of volatility is also significant in capital market. Most of the existing literature demonstrated that the intra-daily volatility exhibits a U-shaped for the market with no break afternoon, such as the exchange market, or doubly U-shaped pattern with one break, such as Japanese stock market (Andersen et al, 2000). Figure 8 shows the autocorrelation of realized volatility, from which the correlation of times-of-16 lags is very significant and declined slowly.

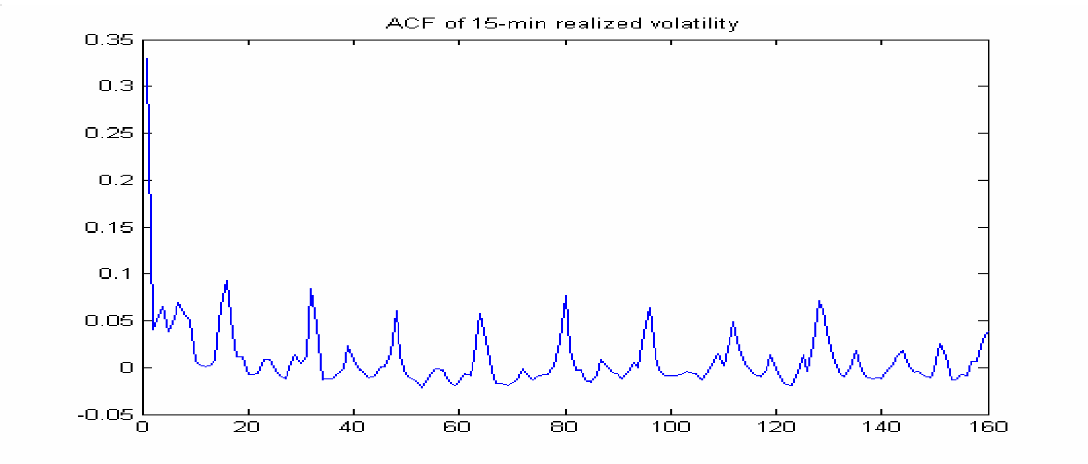

Figure. 8 The autocorrelation of 15-min realized volatility

To demonstrate the intraday pattern of realized volatility deeply, 1-min, 15-min and 30-min average realized volatility are shown in Figure 9.

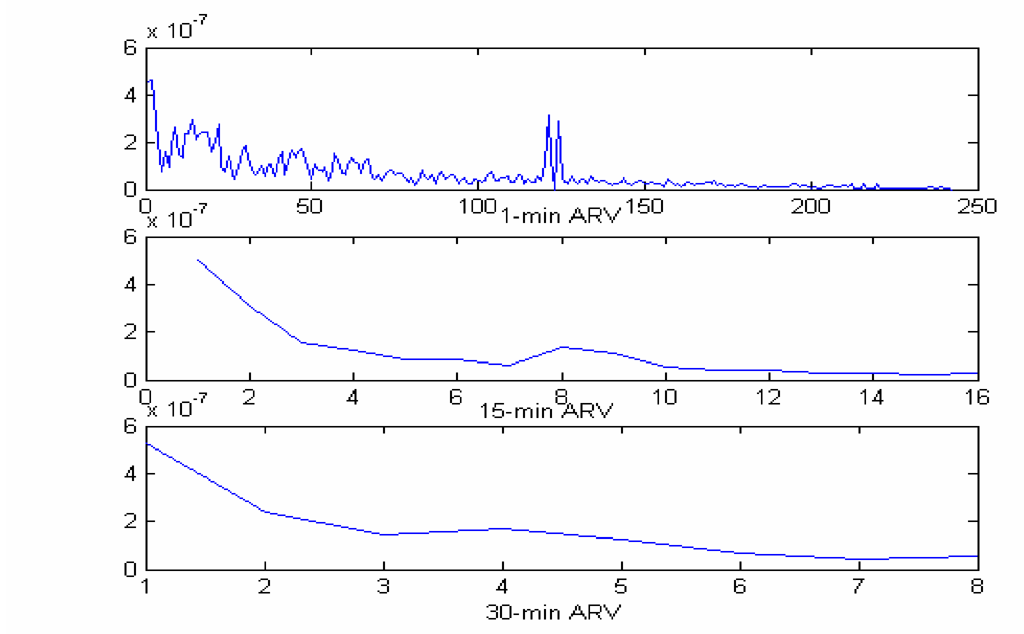

Figure. 9 Average realized volatility (ARV) at different scale

Generally speaking, break of transaction will bring into the accumulation of private and asymmetric information. Longer of the break time probably accumulate more of information un-disposed. Figure.9 provides a rather clearly image about the intraday pattern of realized volatility of CTBM. Volatility is higher in the morning and decline gradually, and it is much lower in afternoon than that in the morning. Of course, as shown in the fist panel of figure.9, volatility goes up before or after the break time in midday. One important reason for the intraday periodicity of CTBM volatility may be the regular release of macroeconomic news, such as the government policy released in the morning or the News Broadcast at previous 7:00 pm in CCTV. It's well known that Chinese Treasury bond market is chiefly influenced by the policy trend of the Chinese government. Most of the important news of China policy is released in the morning or the previous night. For those reasons, it is not strange that the volatility in the morning is high and decrease with time. On the whole, consistent with the microstructure theory, the volatility decreases from morning to afternoon as the disposing of private and asymmetric information of the market, and the intraday pattern of CTBM volatility is more like "L" but not "U" or double "U" shape.

Apart from the intraday pattern of the volatility, the inter-day pattern is also important to be cared for market efficiency or volatility forecasting. Figure.10 is the graph of average realized volatility from Monday to Friday.

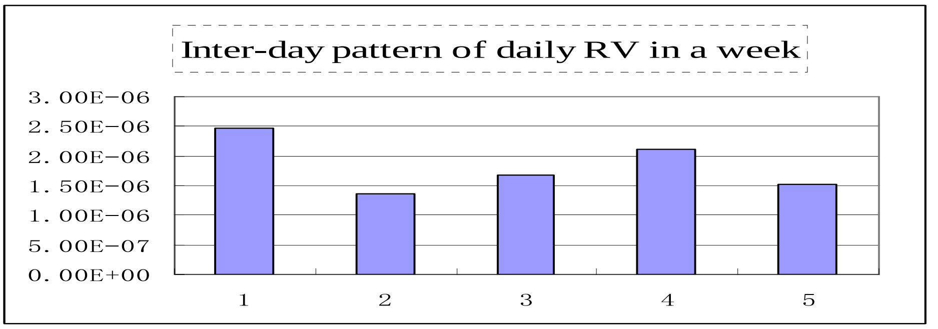

Figure.10 The inter-day pattern of CTBM volatility

It seems that the volatility is highest in Monday and lowest in Tuesday in a week, but the significance test as shown in Table 3 indicates that there is no significant weekly periodic pattern in CTBM.

Table.3 Significance test of inter-day pattern of RV

Day of week	Mon.	Tue.	Wed.	Thu.	Fri.
Mon.	0				
Tue.	0.05316	0			
Wed.	0.13734	0.18811	0		
Thu.	0.014563	-0.054693	-0.1771	0	
Fri.	0.1647	0.24245	0.061988	0.2271	0

3.6 Asymmetry of the volatility

The asymmetry is referred to the return and volatility that positive and negative innovations have different influences on future volatility. The leverage effect theory argues that negative return reduces the value of the company and improves its liability-asset ratio, which increases the risk of the company. That is, negative return has stronger influence on volatility. The relation of return and future volatility (RV and logarithm RV) is shown in Figure 11.

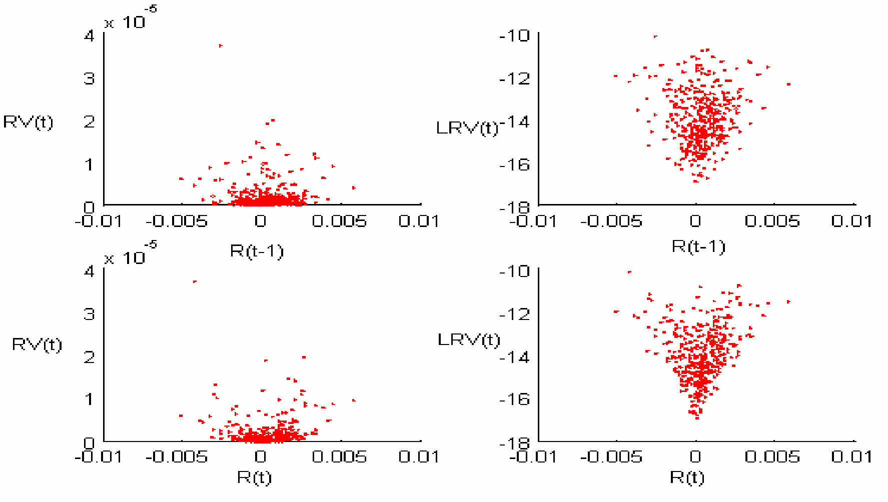

Figure.11 "V"-shape relation between volatility and return

The upper two graphs of Figure.13 describe the relation between return and volatility or logarithm volatility two successive days, and the lower two graphs describe the relation between current return and current volatility. It can be found that volatility and return exhibit a "V"-shape pattern for all the four situations, which is just the suggestion of nonlinear relation between return and volatility. However, nonlinear relation is not equal to asymmetry. If the asymmetry exists, the slop of each side of the "V" shape is equivalent. To test the asymmetry, a econometric model is constructed as,

$$RV_t = \omega + \beta_0 RV_{t-1} + \sum_{i=1}^{p} \alpha_i R^-_{t-i} + \sum_{i=1}^{q} \lambda_i R^+_{t-i} + \varepsilon_t \tag{16}$$

Here, ω, β_0 α_i and λ_i are parameters needed to be estimated. R_t, RV_t and $\ln RV_t$ are daily return, daily RV and logarithm daily RV respectively. p and q are the lag steps represent the influential time of return on volatility. ε_t is the disturbance term with the hypothesis of normal distribution. In addition, to test the asymmetry, a model with the idea of EGARCH (Nelson, 1994) is created as,

$$\ln RV_t^2 = \omega + \beta_0 \ln RV_{t-1}^2 + \sum_{i=1}^{p} \alpha_i \frac{R_{t-i}^-}{\sqrt{RV_{t-i}^2}} + \sum_{i=1}^{p} \lambda_i \frac{R_{t-i}^+}{\sqrt{RV_{t-i}^2}} + \varepsilon_t \tag{17}$$

With different lags of p and q, estimation outputs of model (16) and model (17) are shown in Table 4.

According to the estimation output shown in Table 4, negative α_i and positive λ_i indicate that both too high and too low of the return will lead to higher volatility, which is consistent with "V" shape relation of return and volatility as shown in Figure 11. Moreover, it can be found that the effect of positive return on volatility will persist on two days, but the influence of negative return may only 1 day. As for the asymmetry, the Wald tests (Greene, 2003) of coefficient equality, as shown in Table 5, indicate that there is no significant difference between the two slopes, and impact of negative and positive innovations on volatility is almost the same. That is, the leverage effect or volatility asymmetry is insignificant in CTBM.

Table. 4 Output of model estimation

Coefficient	ω	β_0	α_1	λ_1	α_2	λ_2
Model(16)[a]	0.0008 (0.0000)	0.1267 (0.0343)	-0.2268 (0.0004	0.1627 (0.0036)		
Model(16)[b]	0.0007 (0.0000)	0.0976 (0.1080)	-0.2224 (0.0067)	0.1518 (0.0008)	-0.09 (0.1291)	0.1180 (0.0161)
Model(16)[c]	0.0007 (0.0000)	0.1109 (0.0655)	-0.2454 (0.0002)	0.1523 (0.0066)		0.0914 (0.0460)
Model(17)	-9.4032 (0.0000)	0.3502 (0.0000)	-0.3091 (0.0063)	0.2843 (0.0032)		

Notes: Model(16)[a], model(16)[b] and model(16)[c] are one of the special form of model(16) with different lag steps of p and q. The value in the parentheses is the p-value of the coefficient estimator.

Table. 5 Wald test of the coefficients equality

P-value		Model(18)[a]	Model(18)[b]	Model(18)[c]	Model(19)
H_0	$\alpha_1 = -\lambda_1$	0.27	0.25	0.12	0.81
	$\alpha_2 = -\lambda_2$		0.65		

4. CONCLUSIONS

This paper analyzes the volatility stylized facts of Chinese Treasury bond market based on high frequency data and realized volatility. The main conclusions of this paper are,

4.1 Realized volatility with at 15-minute sampling frequency is rather a good measurement to capture the volatility character of Chinese Treasury bond market, and the conditional distribution of return standardized by realized volatility is nearly normal.

4.2 In Chinese Treasury bond market, the long memory character of volatility is not so significant, the autocorrelation of daily realized volatility walk into the 2 times uncorrelated interval at 5% significance in the third lags.

4.3 The intra-day periodic pattern of return of Chinese

Treasury bond market index is just as two inverse ”Z” in the morning and afternoon transaction time, and that of volatility presents “L” shape. But there is no significant of inter-day periodic pattern in a week for both daily return and volatility.

4.4 The asymmetry relation of return and volatility is not significant in Chinese Treasury bond market, and the upward and downward variation of return have almost the same impact on the current and future volatility. Of course, the nonlinear relation, “V” shape, between return and volatility is significant.

All in all, the stylized facts of Chinese Treasury bond market are different from other stock market or exchange market, as shown in Müller et al (1997), Andersen et al (1999, 2000, 2001, 2003), Corsi et al (2001), Marten (2001), Daconogna (2001), Hol (2002) and so on, including the long-memory character, the asymmetry etc. As for the reason, it is needed to study on it compressively from the angle of microstructure, market efficiency, which is our following work.

REFERENCES

Andersen T G, Bollerslev T, Diefold F X, et al. Exchange rate returns standardized by realized volatility are (nearly) Gaussian [J]. *Multinational Finance Journal*, 2000,4:159-179

Andersen T G, Bollerslev T, Diefold F X, et al. The distribution of realized exchange rate volatility [J]. *Journal of the American statistical association*, 2001, 3:42-55

Andersen T G, Bollerslev T, J Cai. Intraday and interday volatility in the Japanese stock market [J]. *Journal of international finance markets, institutions and money*, 2000 (10): 107-130

Andersen T G, Bollerslev T, Diefold F X, et al. Modeling and forecasting realized volatility [J]. *Econometrica*, 2003(3):579-625

Andersen T G, Bollerslev T, Lange, S, Forecasting Financial Market Volatility: Sample Frequency vis-à-vis Forecast Horizon [J]. *Journal of Empirical Finance*, 1999, 6:457-477.

Baillie R T, Bollerslev T. The message in daily exchange rates: a conditional-variance tale [J]. *Journal of Business & Economic Statistics*, 1989, 7:60-68

Bandi F M, Russell J R, Separating Microstructure Noise from Volatility [J]. *Journal of Financial Economics*, 2006, 79: 655-692.

Bollerslev T, Generalized Autoregressive Conditional Heteroskedasticity [J]. *Journal of Econometrics*, 1986, 31:307-327.

Bollerslev T, Financial Econometrics: Past Developments and Future Challenges [J]. *Journal of Econometrics*, 2000, 100:51-51.

Bollerslev T, Ghysels E, Periodic Autoregressive Conditional Heteroskedasticity [J]. *Journal of Business & Economic Statistics*, 1996, 14(2):139-151.

Braun P A, Nelson D B, Sunier A M, Good News, Bad News, Volatility, and Betas [J]. *Journal of Finance*, 1995, 50(5): 1575-1603.

Corsi F., A Simple Long Memory Model of Realized Volatility. Working Paper. University of Southen Switzerland, 2003.

Corsi F, Zumbach G, Müller U, Dacorogna M. Consistent High-precision Volatility from High Frequency Data [J]. *Economic Notes*, 2001, 30(2): 183-204.

Dacorogna M, Gencay R, Müller U A, et al. *An Introduction to high-frequency finance* [M]. Academic press, USA, 2001

Diamond W, Verrecchia, R E. Constraints on Short-selling and asset price adjustment to private information [J]. *Journal of Financial Economics*, 1987, 18(2):277-311.

Ederington L H, Lee J H. How markets process information: news releases and volatility [J]. *Journal of Finance*, 1993, 48: 1161-1191

Engle R F, Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation [J]. *Econometrica*, 1982, 50(4): 987-1008.

Greene W H. *Econometric Analysis (fifth edition)* [M]. Prentice-Hall International Inc, 2003.

Hol E, Koopman S J. Stock Index Volatility Forecasting with High Frequency Data [J]. *Tingbergen Institute Discussion paper*, TI2002-068/4, 2002.

Martens M, Forecasting Daily Exchange Rate Volatility using Intraday Returns [J]. *Journal of International Money and Finance*, 2001, 20(1):1-23.

Müller U, Dacrogna M, Dave R, et al. Volatilities of different time resolutions-analyzing the dynamics of market components [J]. *Journal of Empirical Finance*, 1997,4(2):213-239

Nelson, D B, Conditional Heteroskedasticity in Asset Returns: a new Approach [J]. *Econometrica*, 1991, 59(2):347-370.

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