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A Game-Theory Analysis of Optimal Leasing and Selling Strategies of Durable Goods in Duopoly Markets

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Abstract

In this paper, we analyze the strategic impact on the choice between leasing and selling in a duopoly market, a generalized model is developed based on the Cournot model, we not only consider the substitution relationship of two firms' new products, but also take the competitiveness between new products and second-hand products into account, and we've discussed six cases respectively under different situations. The results show that if two firms simultaneously employ the same pure strategy, such as (leasing, leasing) or (selling, selling), pure selling strategy turns out to be better than pure leasing strategy. However, if one firm chooses pure selling strategy while the other chooses pure leasing strategy, leasing is the unique dominant behavior. In addition, if one firm's choice is pure strategy, the other's choice is a mix of leasing and selling strategy, pure strategy always dominates the mixed strategy except when the proportion of selling is sufficiently small.

Key words: Durable products; Leasing strategy; Selling strategy

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INTRODUCTION

In the past few decades, selling strategy was always the main way for most of the firms that produced durable goods to market its products and get profits. The related researches also paid much attention to the problems of optimal price or producing quantity. However, in recent years, selling is not the only way for some firms to get profit, a few new marketing strategies becomes very popular, such as leasing strategy and the combination of leasing and selling strategies, which may be of great importance for the firms to get more profits due to the changing market and demand environment.

Taking the automobile industry as an example, after a few years development, the leasing vehicles market has developed very fast and maturely, especially in the North America and Europe. According to the news from American Automobile Manufacturers Association in 2011, the famous leasing companies such as Hertz, Avis, and Zipcar in North America as well as Europcar, Sixt in Europe, annual profit of them has reached to hundreds of millions of euros and the increasing speed is very fast. For example, in the first half of 2009, the profit of Sixt increased by 12% compared with the same period of last year, reached to 82 million euros because of the fast development of leasing industry. In Japan, in order to continue to capture the good opportunity of increasing leasing industry, Toyota launched the Prius plug-in hybrid (PHEV) leasing program in 2010, aiming to create new market area. Based on above information, it is clear that leasing has weakened selling's dominate role in the durable good market, instead, leasing strategy has widely accepted by more and more firms. According to this phenomenon, some researches have studied this kind of problem from several aspects.

Coase (1972) first pointed out that if there were two options (selling and renting) for a monopoly firm to market its durable products, its better choice was leasing instead of selling, because rational consumers would consider that the firm may lower the price in the future after a number of consumers bought the durable goods in the earlier periods. Bulow (1982) built a two-period model for a monopoly firm, the results showed that the profit of selling was obviously less than leasing, the main reason was that leasing strategy allowed the manufacture to increase the price after a period of time. Bucovetsky and Chilton (1986) indicated that monopolist would sell some units or improve the durability of its products to stop the entry of potential competitors when faced with the threat of potential competitors. At this time, selling strategy was much better than leasing. Bulow (1986) then built a model about the choice of durability when the demand was larger than supply, he found that the proportion of leasing and selling depended on the number of firms on the market, he analyzed that a monopolist was willing to improve the durability of new products in order to reduce the products quantity of competitors and increase its profits. Besides, he suggested that the monopoly should employ a mix of selling and leasing strategies for the two periods. Desai and Purohit (1999) developed a two-period model of a duopoly and found that in equilibrium neither firm leased all its units, either they used a mix of leasing and selling strategies or they used only selling strategy. Paul (2000) proved that when we considered a simultaneous move game between two symmetric durable good firms, who had the option to choose between renting and selling before competing in the product market, selling turned out to be the unique dominant behavior of the firms. Agrawal, Ferguson, Toktay, Thomas (2012) found that leasing can be environmentally worse despite remarketing all off-lease products and greener than selling despite the mid-life removal of off-lease products. They also identified when educating consumers to be more environmentally conscious can improve the relative environmental performance of leasing.

In this paper, we also developed a new two-period model in the duopoly environment that evolved from Cournot model, but our model is totally different from previous two-period models, we have generalized our model, which can bring us more realistic and reasonable results. Our most interesting finding was that under any condition in our generalized model, a mix of leasing and selling strategies is never the optimal strategies for the manufacture. Pure strategy is always better than mixed strategy.

1. MODEL DESCRIPTION

In this section, we describe the basic information of our model and explain the assumptions about the product, manufacture and consumer. In order to better understand the marketing issues associated with leasing and selling, we take auto industry as our example.

1.1 Product

The product we consider is a durable, in this paper, in order to express our model and research problems better, we assume that the durable is a car; obviously, other durables such as photocopiers, mainframe, mid-range computers and smart phones and so on are also good examples in durable category. To simplify the analysis, we assume a constant marginal cost, c=0, of producing a car. Because of the nature the durable products, such as car, their life cycle is very long, it seems that multiple periods is more realistic for the model, but in order to make the model more tractable, we assume that the useful life of a car can last two periods. Actually, the amount of time is not crucial, it is important to assume the life of cars lasts more than one period and they have finite lives. In addition, we assume that there are two types of car available in the market: new cars and used cars. In the first period, there are only new cars existing in the market, in the second period, there are both new cars and used cars on the market. Because new cars are produced and marketed both in the beginning of the first period and second period, the cars that are transferred from the first period will become used cars in the second period.

1.2 Manufacture

There are two manufacturers who manufacture and market the identical products in the market with a constant marginal cost of c. because the presence of positive costs does not affect the nature of our results, we set c=0. In addition, the strategies that the manufacturers can choose are pure leasing, pure selling and the combination of leasing and selling. In the first period, the manufacturers sell or lease their new cars while in the second period, we assume that they only choose to sell the new and used cars. This assumption is reasonable because in order to adapt to the diverse and changing market demand, the manufacturers will produce new car models instead of always producing the same old car model after a period of time's "selling", so in the second period, although the current old car model still produces and sells, the manufacturers would rather sell all of them than lease the out of fashion cars, because at this time, consumers absolutely prefer new car model than the old ones. If the manufacturers continue to lease current type of car model, after being returned from the consumers at the end of contract period, the old car model is hardly been sold or disposed due to its out of fashion nature, which is bad for the manufacturers total profit. According to above reasons, the manufactures will only choose to sell their new and old products in the second period.

1.3 CONSUMER

We assume that every consumer only buys one car and consumers who buy the new cars in the first period will hold them for two periods that mean they have the ownership of the cars for two periods. The consumers who choose to lease the new cars in the first period will return the cars to the manufacturers at the end of period one. In this paper, all the consumers we refer to are indifferent between buying and leasing and we also don't divide the consumers into rational consumers and myopic consumers just like some other papers, consumers choose the way the consume only by the economic conditions and personal preference.

Before we analyze our model, some notations should be provided. Table 1 explains the meaning of each notation.

Table 1
Summary of Notations

Parameter	Specific meaning
a	Capacity of the market
p^k_{ij}	Price of selling a unit of product, i=1,2, j=new, used, k=firm A,B
l^k_{ij}	Price of leasing a unit of product, i=1,2, j=new, used, k=firm A,B
q^k_{ij}	Quantity of products that manufacturers produced, i=1,2, j=new, used, k=firm A,B
δ	Substitute coefficient of leasing and selling price in the first period
γ	Substitute coefficient of price of new product and used product in the second period
π^{mn}_{ik}	Firm's profit in the second period, i=1,2, k=firm A,B, m=selling, leasing, combination, n=selling, leasing, combination.
π^{mn}_{k}	Total profit of two periods
f	proportion of selling

2. ANALYSIS OF MODEL

There are two manufacturers who manufacture and market the identical products in the market. The strategies that the manufacturers can choose are pure leasing, pure selling and the combination of leasing and selling. Two manufacturers compete with each other and they have nine kinds of strategy profile, but three of strategy combinations are same as other cases, therefore, there are only six kinds of strategy profile, which are depicted in Table 2. Note that we use "mix" stands for the combination of leasing and selling strategy.

Table 2
Strategy Profile of the Two Firms

	Case1	Case 2	Case 3	Case 4	Case 5	Case 6
Firm A	Selling	Selling	Selling	Leasing	Leasing	Mix
Firm B	Selling	Leasing	Mix	Leasing	Mix	Mix

2.1 Firm *A* Selling and Firm *B* Selling

In this case, we analyze the case that firm A chooses pure selling strategy and firm B also chooses pure selling strategy. Then the demand functions of *A*, *B* firms in two periods are given by:

$$\begin{cases} q^A_{1n} = a - b_1 p^A_{1n} + b_2 p^B_{1n} \\ q^B_{1n} = a - b_1 p^B_{1n} + b_2 p^A_{1n} \end{cases}, \tag{1}$$

$$\begin{cases} q^A_{2n} = (a - q^A_{1n} - q^B_{1n}) - d_1 p^A_{2n} + d_2 p^B_{2n} \\ q^B_{2n} = (a - q^A_{1n} - q^B_{1n}) - d_1 p^B_{2n} + d_2 p^A_{2n} \end{cases}. \tag{2}$$

From the Equations (1) and (2), we can obtain the price p^A_{1n}, p^B_{1n}, p^A_{2n}, p^B_{2n}, respectively.

$$p^A_{1n} = \frac{a(b_1 + b_2) - b_1 q^A_{1n} - b_2 q^B_{1n}}{b_1^2 - b_2^2},$$

$$p^B_{1n} = \frac{a(b_1 + b_2) - b_2 q^A_{1n} - b_1 q^B_{1n}}{b_1^2 - b_2^2},$$

$$p^A_{2n} = \frac{a(d_1+d_2) - (d_1+d_2)q^A_{1n} - (d_1+d_2)q^B_{1n} - d_1 q^A_{2n} - d_2 q^B_{2n}}{d_1^2 - d_2^2},$$

$$p^B_{2n} = \frac{a(d_1 + d_2) - (d_1 + d_2)q^A_{1n} - (d_1 + d_2)q^B_{1n} - d_2 q^A_{2n} - d_1 q^B_{2n}}{d_1^2 - d_2^2}.$$

The second period profit of firm A is $\pi^{SS}_{2A}=p^A_{2n}q^A_{2n}$, the firm chooses optimal quantity q^{*A}_{2n} to maximize its second period profit by solving the first order condition. The second period profit of firm B is $\pi^{SS}_{2B}=p^B_{2n}q^B_{2n}$ the firm chooses optimal quantity q^{*B}_{2n} to maximize its second period profit by solving the first order condition. Solving both firms' problems simultaneously, q^*_{2A} yields q^*_{2B}.

Then we consider the first period problems. By solving the first order condition, firm A chooses optimal quantity q^{*A}_{1n} to maximize its total profit of two periods: $\pi^{SS}_{A}=\pi^{SS}_{1A}\pi^{*SS}_{2A}$, note that $\pi^{SS}_{1A}=p^A_{1n}q^A_{1n}$. Meanwhile, firm B chooses optimal quantity q^{*B}_{1n} to maximize its total profit of two periods: $\pi^{SS}_{B}=\pi^{SS}_{1B}\pi^{*SS}_{2B}$, note that $\pi^{SS}_{1B}=p^B_{1n}q^B_{1n}$.

Solving both firms' problems simultaneously yields q^*_{1A} and q^*_{1B}. Because the final expressions of q^*_{1A} and q^*_{1B} are too long (the other five cases are also the same situation), we didn't show them here. In order to capture the important managerial insights, we will use the numerical study to reflect important results of our model.

2.2 Firm *A* Selling and Firm *B* Leasing

In this case, firm *A* chooses pure selling strategy while firm B chooses pure leasing strategy. Because $\gamma \in (0,1)$, is the substitute coefficient of price of new product and used product in the second period, so we reasonably assume that $p^A_{2u}=\gamma p^A_{2n}$, $p^B_{2u}=\gamma p^B_{2n}$. Note that this assumption also suits for the following four cases. Demand functions of *A*, *B* firms in two periods are given by:

$$\begin{cases} q^A_{1n} = a - b_1 p^A_{1n} + c_2 l^B_{1n} \\ q^B_{1n} = a - c_1 l^B_{1n} + b_2 p^A_{1n} \end{cases}, \tag{3}$$

$$\begin{cases} q^A_{2n} = (a - q^A_{1n}) - d_1 p^A_{2n} + d_2 p^B_{2n} + d_4 p^B_{2u} \\ q^B_{2n} = (a - q^A_{1n}) - d_1 p^B_{2n} + d_2 p^A_{2n} + d_3 p^B_{2u} \end{cases}. \tag{4}$$

Then we can obtain the price p^A_{1n}, p^B_{1n}, p^A_{2n}, p^B_{2n}, respectively from Equations (3) and (4).

$$p_{1n}^{A}=\frac{ac_1+ac_2-c_1q_{1n}^{A}-c_2q_{1n}^{B}}{b_1c_1-b_2c_2},\qquad l_{1n}^{B}=\frac{ab_1+ab_2-b_2q_{1n}^{A}-b_1q_{1n}^{B}}{b_1c_1-b_2c_2},$$

$$p_{2n}^{A}=\frac{ad_1-ad_3\gamma-(d_1-d_3\gamma+d_2+d_4\gamma)q_{1n}^{A}-(d_1-d_3\gamma)q_{2n}^{A}-(d_2+d_4\gamma)q_{2n}^{B}}{d_1^2-d_2^2-d_1d_3\gamma-d_2d_4\gamma},$$

$$p_{2n}^{B}=\frac{ad_1+ad_2-(d_1+d_2)q_{1n}^{A}-d_2q_{2n}^{A}-d_1q_{2n}^{B}}{d_1^2-d_2^2-d_1d_3\gamma-d_2d_4\gamma}.$$

The second period profit of firm A is $\pi_{2A}^{SL}=p_{2n}^{A}q_{2n}^{A}$ the firm chooses optimal quantity q_{2n}^{*A} to maximize its second period profit by solving the first order condition. The second period profit of firm B is $\pi_{2B}^{SL}=p_{2n}^{B}q_{2n}^{B}+p_{2u}^{B}q_{1n}^{B}$ the firm chooses optimal quantity q_{2n}^{*B} to maximize its second period profit by solving the first order condition. Solving both firms' problems simultaneously, yields q_{2A}^{*} and q_{2B}^{*}.

Then we consider the first period problems. By solving the first order condition, firm A chooses optimal quantity q_{1n}^{*A} to maximize its total profit of two periods: $\pi_{A}^{SL}=\pi_{1A}^{SL}+\pi_{2A}^{*SL}$, note that $\pi_{1A}^{SL}=p_{1n}^{A}q_{1n}^{A}$. Meanwhile, firm B chooses optimal quantity q_{1n}^{*B} to maximize its total profit of two periods: $\pi_{B}^{SL}=\pi_{1B}^{SL}+\pi_{2B}^{*SL}$, note that $\pi_{1B}^{SL}=l_{1n}^{B}q_{1n}^{B}$. Solving both firms' problems simultaneously yields q_{1A}^{*} and q_{1B}^{*}.

As shown in Figure 1, the revenue of firm B, who chooses pure leasing strategy, is greater than that of firm A. This result illustrates if one firm chooses pure selling strategy and the other firm chooses pure leasing strategy, the pure leasing strategy always dominates pure selling strategy when the price between new products and second-hand products is closer.

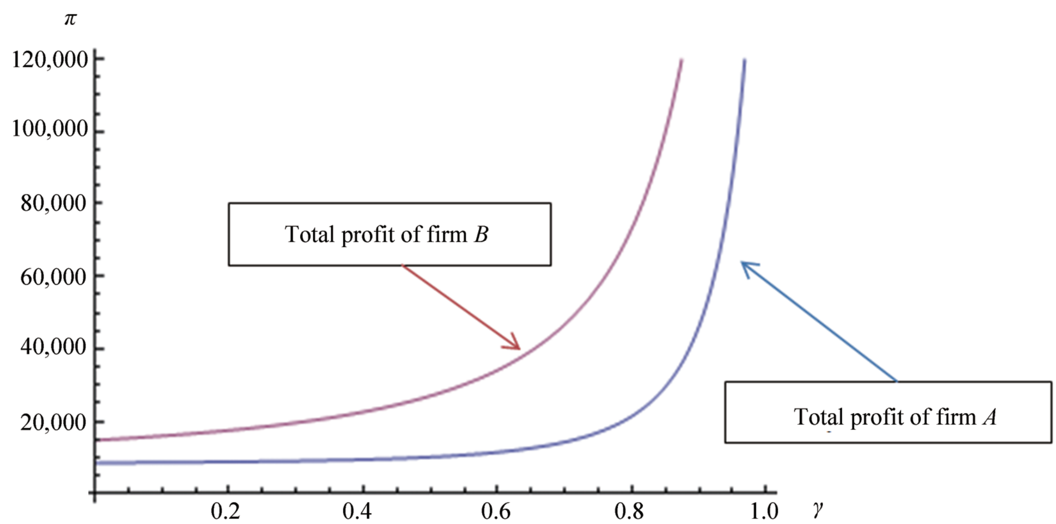

Figure 1
Manufacturers' Profit With Changes
Note. a=100, b_1=1, b_2=0.9, c_1=1, c_2=0.9, d_1=1, d_2=0.9, d_3=d_4=0.3.

We can also see that as the γ becomes larger, both firms revenue increases. The reason is that high second-hand price leads to the increasing demand of new products and the price of new products is much higher than second-hand price, so the profit of the firm will increase with the larger γ.

2.3 Firm *A* Selling and Firm *B* Both Leasing and Selling

In this case, firm A chooses pure selling strategy while firm B chooses combination strategy. Note that $\delta\in(0,1)$ is the substitute coefficient between lasing price and selling price. We assume $l_{1n}^{B}=\delta p_{1n}^{B}$ is reasonable because leasing price is always smaller than selling price in practical, f stands for the proportion that manufacturer sells his products. Demand functions of A, B firms in two periods are given by:

$$\begin{cases} q_{1n}^{A}=a-b_1p_{1n}^{A}+b_2p_{1n}^{B}+c_2l_{1n}^{B} \\ q_{1n}^{B}=a-b_1p_{1n}^{B}-c_1l_{1n}^{B}+b_2p_{1n}^{A} \end{cases}, \tag{5}$$

$$\begin{cases} q_{2n}^{A}=(a-q_{1n}^{A}-fq_{1n}^{B})-d_1p_{2n}^{A}+d_2p_{2n}^{B}+d_4p_{2u}^{B} \\ q_{2n}^{B}=(a-q_{1n}^{A}-fq_{1n}^{B})-d_1p_{2n}^{B}+d_2p_{2n}^{A}+d_3p_{2u}^{B} \end{cases}. \tag{6}$$

Then we can obtain the price p_{1n}^{A}, p_{1n}^{B}, p_{2n}^{A}, p_{2n}^{B}, respectively from Equations (5) and (6).

$$p_{1n}^{A} = \frac{a[b_1 + b_2 + \delta(c_1 + c_2)] - (b_1 + c_1\delta)q_{1n}^{A} - (b_2 + c_2\delta)q_{1n}^{B}}{b_1^2 + \delta b_1 c_1 - b_2^2 - \delta b_2 c_2},$$

$$p_{1n}^{B} = \frac{a(b_1 + b_2) - b_2 q_{1n}^{A} - b_1 q_{1n}^{B}}{b_1^2 + \delta b_1 c_1 - b_2^2 - \delta b_2 c_2},$$

$$p_{2n}^{A} = \frac{a(d_1 - d_3\gamma + d_2 + d_4\gamma) - (d_1 - d_3\gamma + d_2 + d_4\gamma)q_{1n}^{A} - f(d_1 - d_3\gamma + d_2 + d_4\gamma)q_{1n}^{B}}{d_1^2 - d_2^2 - d_1 d_3\gamma - d_2 d_4\gamma} - \frac{(d_1 - d_3\gamma)q_{2n}^{A} - (d_2 + d_4\gamma)q_{2n}^{B}}{d_1^2 - d_2^2 - d_1 d_3\gamma - d_2 d_4\gamma},$$

$$p_{2n}^{B} = \frac{a(d_1 + d_2) - (d_1 + d_2)q_{1n}^{A} - f(d_1 + d_2)q_{1n}^{A} - d_2 q_{2n}^{A} - d_1 q_{2n}^{B}}{d_1^2 - d_2^2 - d_1 d_3\gamma - d_2 d_4\gamma}.$$

The second period profit of firm A is $\pi_{2A}^{SC}=p_{2n}^{A}q_{2n}^{A}$, the firm chooses optimal quantity q_{2n}^{*A} to maximize its second period profit by solving the first order condition. The second period profit of firm B is $\pi_{2B}^{SC}=p_{2n}^{B}q_{2n}^{B}+p_{2u}^{B}(1\text{-}f)q_{1n}^{B}$, the firm chooses optimal quantity q_{2n}^{*B} to maximize its second period profit by solving the first order condition. Solving both firms' problems simultaneously, yields q_{2A}^{*} and q_{2B}^{*}.

Then we consider the first period problems. By solving the first order condition, firm A chooses optimal quantity q_{1n}^{*A} to maximize its total profit of two periods: $\pi_{A}^{SC}=\pi_{1A}^{SC}+\pi_{2A}^{*SC}$, note that $\pi_{1A}^{SC}=p_{1n}^{A}q_{1n}^{A}$. Meanwhile, firm B chooses optimal quantity q_{1n}^{*B} to maximize its total profit of two periods: $\pi_{B}^{SC}=\pi_{1B}^{SC}+\pi_{2B}^{*SC}$, note that $\pi_{1B}^{SS}=l_{1n}^{B}(1\text{-}f)q_{1n}^{B}+p_{1n}^{B}fq_{1n}^{B}$.

Solving both firms' problems simultaneously yields q_{1A}^{*} and q_{1B}^{*}.

As shown in Figure 2, if one firm chooses pure selling strategy and the other firm chooses a mix of the leasing and selling strategies, pure selling strategy always dominates the mixed strategy as the leasing price is approaching to the selling price.

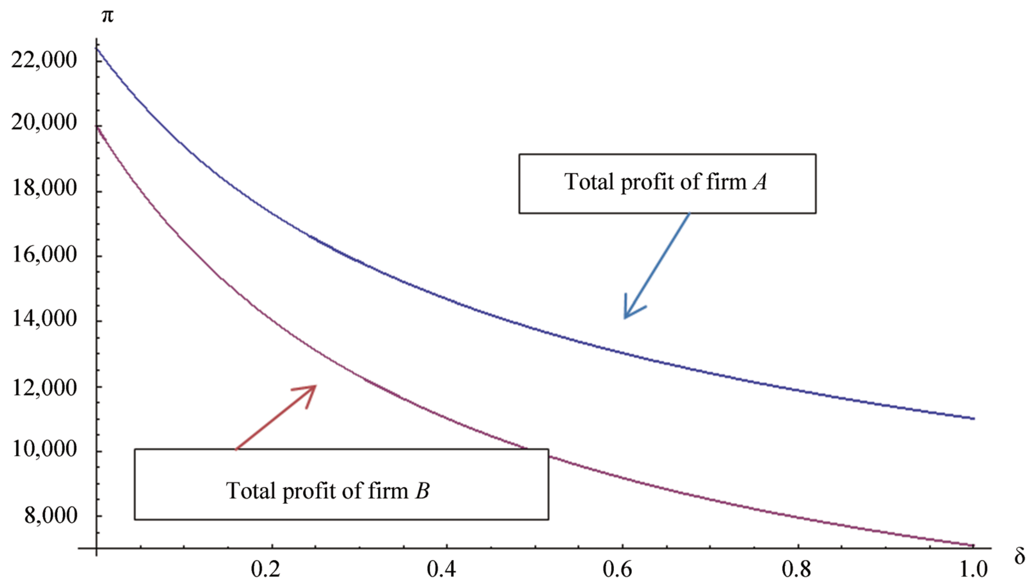

Figure 2
Manufacturers' Profit With Changes
Note. a=100, b_1=1, b_2=0.9, c_1=1, c_2=0.9, d_1=1, d_2=0.9, d_3=d_4=0.3, f=0.5, γ=0.7

The figure displays the manufactures' revenue under this case, we can see that with the δ increases, the profits of both firms to decreases, it because if δ is close to 1, the leasing price is close to selling price , then the quantity of firm B's products becomes less, the profit of firm B in the first period will decrease ,when it comes to second period, the quantity of firm B's second-hand products also decreases, so the second period's profit also becomes less, the total profit ,therefore, declines with becomes larger. As for firm A, firm B's higher leasing price leads to the larger demand for its new products in the first period, then the price of its new products will relatively drop, although the selling quantity increases, the long-term profit may decrease.

As shown in Figure 3, if one firm chooses pure selling strategy and the other firm chooses a mix of the leasing and selling strategies, when the proportion of selling is relatively small, the mixed strategy is better than pure selling strategy. As the proportion of selling becomes larger and closes to 1, pure selling strategy is better than mixed strategy.

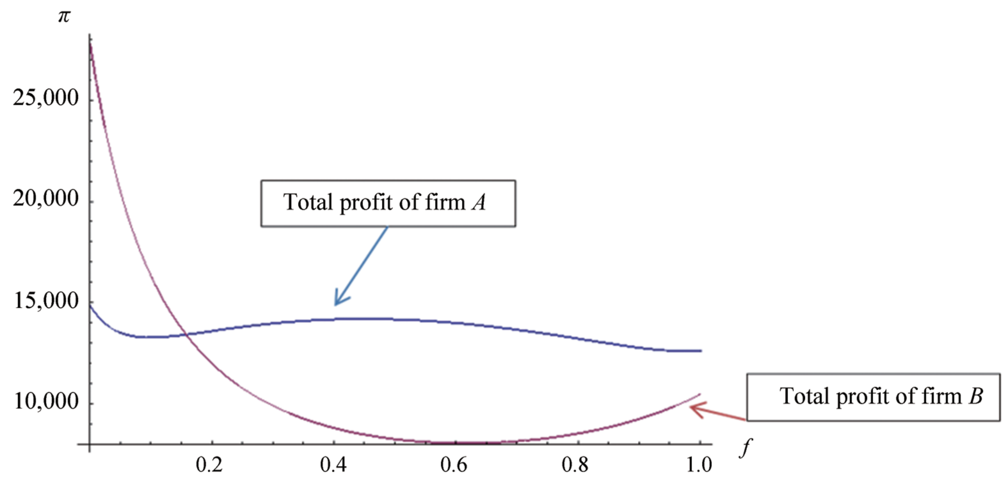

Figure 3
Manufacturers' Profit With Changes
Note. a=100, b_1=1, b_2=0.9, c_1=1, c_2=0.9, d_1=1, d_2=0.9, d_3=d_4=0.3, γ=0.7, δ=0.6.

The figure displays the manufactures' profits under this case under this case; we can see that if one firm chooses to sell all its products and the other firm chooses to combine the leasing and selling strategies, both selling and leasing strategies dominates pure selling strategy when the f is relatively small. As for firm *B*, the smaller the *f*, the higher the p^B_{2n} and p^B_{2u} so the profit in the second period, so the total profit is high when the f is very small.

2.4 Firm *A* Leasing and Firm *B* Leasing

In this case, firm *A* chooses pure selling strategy while firm B also chooses leasing strategy. Demand functions of *A*, *B* firms in two periods are given by:

$$\begin{cases} q^A_{1n} = a - c_1 l^A_{1n} + c_2 l^B_{1n} \\ q^B_{1n} = a - c_1 l^B_{1n} + c_2 l^A_{1n} \end{cases}, \qquad (7)$$

$$\begin{cases} q^A_{2n} = a - d_1 p^A_{2n} + d_2 p^B_{2n} + d_3 p^A_{2u} + d_4 p^B_{2u} \\ q^B_{2n} = a - d_1 p^B_{2n} + d_2 p^A_{2n} + d_3 p^B_{2u} + d_4 p^A_{2u} \end{cases}. \qquad (8)$$

Then we can obtain the price l^A_{1n}, l^B_{1n}, p^A_{2n}, p^B_{2n} respectively from Equations (7) and (8).

$$l^A_{1n} = \frac{a(c_1+c_2) - c_1 q^A_{1n} - c_2 q^B_{1n}}{c_1^2 - c_2^2}\, l^B_{1n} = \frac{a(c_1+c_2) - c_1 q^B_{1n} - c_2 q^A_{1n}}{c_1^2 - c_2^2},$$

$$p^A_{2n} = \frac{a(d_1 - d_3\gamma + d_2 + d_4\gamma) - (d_1 - d_3\gamma) q^A_{2n} - (d_2 + d_4\gamma) q^B_{2n}}{(d_1 - d_3\gamma)^2 - (d_2 + d_4\gamma)^2},$$

$$p^B_{2n} = \frac{a(d_1 - d_3\gamma + d_2 + d_4\gamma) - (d_1 - d_3\gamma) q^B_{2n} - (d_2 + d_4\gamma) q^A_{2n}}{(d_1 - d_3\gamma)^2 - (d_2 + d_4\gamma)^2},$$

The second period profit of firm *A* is $\pi^{LL}_{2A}=p^A_{2n}q^A_{2n}+p^A_{2u}q^A_{1n}$, the firm chooses optimal quantity q^{*A}_{2n} to maximize its second period profit by solving the first order condition. The second period profit of firm *B* is $\pi^{LL}_{2B}=p^B_{2n}q^B_{2n}+p^B_{2u}q^B_{1n}$, the firm chooses optimal quantity q^{*B}_{2n} to maximize its second period profit by solving the first order condition. Solving both firms' problems simultaneously, yields q^*_{2A} and q^*_{2B}.

Then we consider the first period problems. By solving the first order condition firm A chooses optimal quantity q^{*A}_{1n} to maximize its total profit of two periods: $\pi^{LL}_A=\pi^{LL}_{1A}+\pi^{*LL}_{2A}$, note that $\pi^{LL}_{1A}=l^A_{1n}q^A_{1n}$. Meanwhile, firm *B* chooses optimal quantity q^{*B}_{1n} to maximize its total profit of two periods: $\pi^{LL}_B=\pi^{LL}_{1B}+\pi^{*LL}_{2B}$, note that $\pi^{LL}_{1B}=l^B_{1n}q^B_{1n}$.

Solving both firms' problems simultaneously yields q^*_{1A} and q^*_{1B}.

As shown in Figure 4, when two firms simultaneously choose leasing strategy, the closer between second hand price and new product price in the second period, the more profit they will get. It should be emphasized that when two firms choose same strategy, their profit (the competing equilibrium outcomes) are the same.

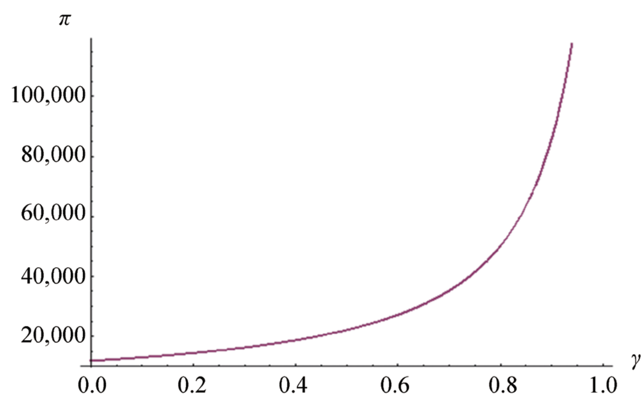

Figure 4
Manufacturers' Profit With Changes
Note. a=100, b_1=1, b_2=0.9, c_1=1, c_2=0.9, d_1=1, d_2=0.9, d_3=d_4=0.3.

The figure displays the manufactures' profits under this case under this case, we can see that as the γ becomes larger, the profit of the firms is increasing, it because larger γ leads to higher price of second-hand products, which in turn increase the demand for new products, so the total profit may increase .

2.5 Firm *A* Leasing and Firm *B* Both Leasing and Selling

In this case, firm *A* chooses pure leasing strategy while firm *B* chooses both leasing and strategy. Demand functions of A, B firms in two periods are given by:

$$\begin{cases} q_{1n}^A = a - c_1 l_{1n}^A + c_2 l_{1n}^B + b_2 p_{1n}^B \\ q_{1n}^B = a - b_1 p_{1n}^B - c_1 l_{1n}^B + c_2 l_{1n}^A \end{cases}, \quad (9)$$

$$\begin{cases} q_{2n}^A = (a - f q_{1n}^B) - d_1 p_{2n}^A + d_2 p_{2n}^B + d_3 p_{2u}^A + d_4 p_{2u}^B \\ q_{2n}^B = (a - f q_{1n}^B) - d_1 p_{2n}^B + d_2 p_{2n}^A + d_3 p_{2u}^B + d_4 p_{2u}^A \end{cases}. \quad (10)$$

Then we can obtain the price l_{1n}^A, l_{1n}^B, p_{2n}^A, p_{2n}^B respectively from Equations (9) and (10).

$$l_{1n}^A = \frac{a(b_1 + b_2 + c_1\delta + c_2\delta) - (b_1 + c_1\delta) q_{1n}^A - (b_2 + c_2\delta) q_{1n}^B}{b_1 c_1 - b_2 c_2 + c_1^2\delta - c_2^2\delta},$$

$$p_{1n}^B = \frac{a(c_1 + c_2) - c_2 q_{1n}^A - c_1 q_{1n}^B}{b_1 c_1 - b_2 c_2 + c_1^2\delta - c_2^2\delta},$$

$$l_{1n}^B = \frac{a\delta(c_1 + c_2) - c_2 q_{1n}^A - c_1 q_{1n}^B}{b_1 c_1 - b_2 c_2 + c_1^2\delta - c_2^2\delta},$$

$$p_{2n}^A = \frac{a(d_1 - d_3\gamma + d_2 + d_4\gamma) - f(d_1 - d_3\gamma + d_2 + d_4\gamma) q_{1n}^B - (d_1 - d_3\gamma) q_{2n}^A - (d_2 + d_4\gamma) q_{2n}^B}{(d_1 - d_3\gamma)^2 - (d_2 + d_4\gamma)^2},$$

$$p_{2n}^B = \frac{a(d_1 - d_3\gamma + d_2 + d_4\gamma) - f(d_1 - d_3\gamma + d_2 + d_4\gamma) q_{1n}^B - (d_2 + d_4\gamma) q_{2n}^A - (d_1 - d_3\gamma) q_{2n}^B}{(d_1 - d_3\gamma)^2 - (d_2 + d_4\gamma)^2}.$$

The second period profit of firm A is $\pi_{2A}^{LC} = p_{2n}^A q_{2n}^A + p_{2u}^A q_{1n}^A$, the firm chooses optimal quantity q_{2n}^{*A} to maximize its second period profit by solving the first order condition. The second period profit of firm B is $\pi_{2B}^{LC} = p_{2n}^A q_{2n}^A + p_{2u}^A (1-f) q_{1n}^B$, the firm chooses optimal quantity q_{2n}^{*B} to maximize its second period profit by solving the first order condition. Solving both firms' problems simultaneously, yields q_{2A}^* and q_{2B}^*.

Then we consider the first period problems. By solving the first order condition firm A chooses optimal quantity q_{1n}^{*A} to maximize its total profit of two periods: $\pi_A^{LC} = \pi_{1A}^{LC} = \pi_{2A}^{*LC}$, note that $\pi_{1A}^{LC} = l_{1n}^A q_{1n}^A$. Meanwhile, firm B chooses optimal quantity q_{1n}^{*B} to maximize its total profit of two periods: $\pi_B^{LC} = \pi_{1B}^{LC} = \pi_{2B}^{*LC}$, note that $\pi_{1B}^{LC} = l_{1n}^B (1-f) q_{1n}^B + p_{1n}^B f q_{1n}^B$.

Solving both firms' problems simultaneously yields q_{1A}^* and q_{1B}^*.

As shown in Figure 5, if one firm chooses pure leasing strategy and the other firm chooses a mix of the leasing and selling strategies, pure leasing strategy always dominates the mixed strategy as the second hand price is approaching to the selling price.

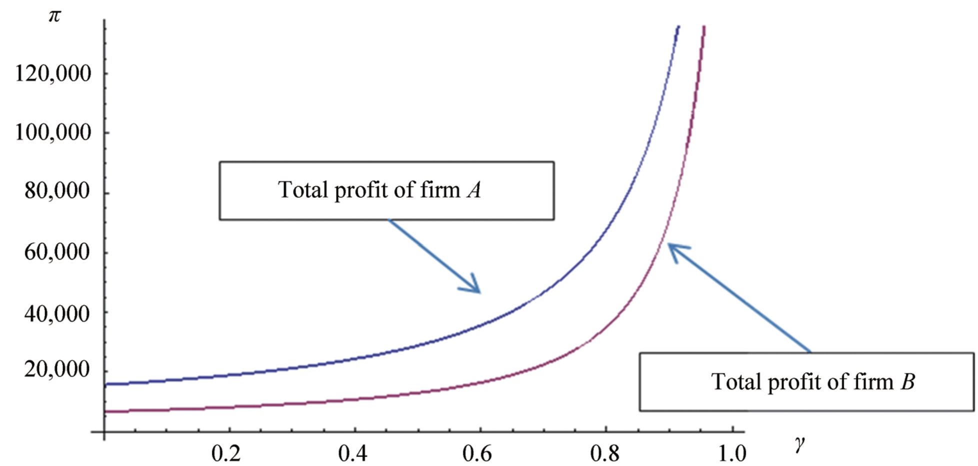

Figure 5
Manufacturers' Profit With Changes
Note. a=100, b_1=1, b_2=0.9, c_1=1, c_2=0.9, d_1=1, d_2=0.9, d_3=d_4=0.3, γ=0.7, δ=0.6.

The results also show that as the γ is close to 1, both of the two firms' profit are increasing, this because the second-hand products 'price becomes high, which means the demand of new products in the second period will increase, the total profit of both firms will increase with the larger γ.

2.6 Both Firm *A* and Firm *B* Adopt Combination Strategy

In this case, firm A chooses combination strategy while firm B also chooses combination strategy. Demand functions of A, B firms in two periods are given by:

$$\begin{cases} q_{1n}^A = a - c_1 l_{1n}^A - b_1 p_{1n}^A + c_2 l_{1n}^B + b_2 p_{1n}^B \\ q_{1n}^B = a - b_1 p_{1n}^B - c_1 l_{1n}^B + c_2 l_{1n}^A + + b_2 p_{1n}^A \end{cases}, \quad (11)$$

$$\begin{cases} q_{2n}^A = (a - h q_{1n}^A - f q_{1n}^B) - d_1 p_{2n}^A + d_2 p_{2n}^B + d_3 p_{2u}^A + d_4 p_{2u}^B \\ q_{2n}^B = (a - h q_{1n}^A - f q_{1n}^B) - d_1 p_{2n}^B + d_2 p_{2n}^A + d_3 p_{2u}^B + d_4 p_{2u}^A \end{cases}. \quad (12)$$

Just as the same solution process of five previous cases, we can deduce l_{1n}^A, l_{1n}^B, p_{1n}^A, p_{1n}^B, p_{2n}^A, p_{2n}^B from

Equations (11) and (12), here we won't repeat the process again.

The second period profit of firm *A* is $\pi_{2A}^{CC}=p_{2n}^{A}q_{2n}^{A}+p_{2u}^{A}(1\text{-}f)q_{1n}^{A}$, the firm chooses optimal quantity q_{2n}^{*A} to maximize its second period profit by solving the first order condition. The second period profit of firm B is $\pi_{2B}^{CC}=p_{2n}^{B}q_{2n}^{B}+q_{2u}^{B}(1\text{-}f)p_{1n}^{B}$, the firm chooses optimal quantity q_{2n}^{*B} to maximize its second period profit by solving the first order condition. Solving both firms' problems simultaneously, yields q_{2A}^{*} and q_{2B}^{*}.

Then we consider the first period problems. By solving the first order condition firm *A* chooses optimal quantity q_{1n}^{*A} to maximize its total profit of two periods: $\pi_{A}^{CC}=\pi_{1A}^{SC}+\pi_{2A}^{*SC}$, note that $\pi_{1A}^{CC}=l_{1n}^{A}(1\text{-}f)q_{1n}^{A}+p_{1n}^{A}f\,q_{1n}^{A}$. Meanwhile, firm B chooses optimal quantity q_{1n}^{*B} to maximize its total profit of two periods: $\pi_{B}^{SC}=\pi_{1B}^{SC}+\pi_{2B}^{*SC}$, note that $\pi_{1B}^{SS}=l_{1n}^{B}(1\text{-}f)q_{1n}^{B}+p_{1n}^{B}fq_{1n}^{B}$.

Solving both firms' problems simultaneously yields q_{1A}^{*} and q_{1B}^{*}.

3. COMPARISON OF CASES

This part, we compared six cases based on the same parameter values set in chapter 3. We take firm *A*'s profit as an example to express his optimal profit under every case.

Table 3
Firm A's Profit With the Market Capacity Increase

(*A, B*)	20	40	60	80	100
(sell, sell)	897.02	3588.08	8073.18	14352.30	22425.50
(sell, lease)	406.50	1626.02	3658.54	6504.06	10162.60
(sell, mix)	520.94	2083.74	4688.42	8334.98	13023.40
(lease, lease)	885.19	3540.75	7966.69	14163.01	22129.70
(lease, mix)	8540.56	34162.20	76865.04	136648.96	213514.00
(mix, mix)	213.51	854.04	1921.59	3416.17	5337.76

Table 3 shows the changes of firm *A*'s profit under six cases when the market capacity increases. From this table, it is interesting to find that if both of two firms choose combination of leasing and selling strategy, they will get the least profit compared with the other five cases. What's more, if firm *A* chooses pure leasing strategy while firm *B* chooses a mix of two strategies, firm *A* will achieve highest profit compared with the other five cases. But this results raise another issue, if firm *A* can achieve higher profit under this case, firm B will imitates firm A's pure leasing strategy and give up mixed strategy, finally, both firm will adopt pure leasing strategy. However, as is shown in Table 2, (lease and lease) strategy is not the best choice because of lower profit compared with other cases. This phenomenon of our results perfectly matches the prisoner dilemma.

CONCLUSION

In this paper, we establish a game model to investigate the strategic impact on the choice between leasing strategy and selling strategy in a duopoly market. Both the substitution of two firms' new products and competitiveness of new and old products are taken into account. According to our numerical results, some significant results are found. First, if two firms simultaneously employ the same pure strategy, pure selling strategy turns out to be better than pure leasing strategy. Second, if one firm chooses pure selling strategy while the other chooses pure leasing strategy, leasing strategy always dominates selling strategy. Third, if one firm's choice is pure strategy, the other's choice is a mix of leasing and selling strategy, pure strategy always dominates the mixed strategy except when the proportion of selling is sufficiently small. Finally, our numerical results also proved the insights of "prison dilemma".

Although we believe that our model helps to explain how different strategies can alter the optimal decisions of firms. First, it is not without its limitations. In reality, we know that the lifecycle of a durable good lasts more than two periods. Second, in the future research, multiple competitors should be considered. Work is ongoing to investigate the problem we didn't consider in this paper, we hope our theoretical work in this paper will help firm to make better decision.

REFERENCES

Agrawal, V. V., Ferguson, M., Toktay, L. B., & Valerie, M. T. (2012). Is leasing greener than selling? *Management Science, 58*(3), 523-533.

Blanchard, O., & Melino, A. (1986). The cyclical behavior of prices and quantities: The case of the automobile market. *Journal of Monetary Economics, 17*(3), 379-407.

Coase, R. H. (1972). Durability and monopoly. *Journal of Law and Economics, 15*(1), 143-149.

Dasu, S., & Tong, C. (2010). Dynamic pricing when consumers are strategic: Analysis of posted and contingent pricing schemes. *Eur. J. Oper. Res, 204*(3), 662-671.

Elmaghraby, W., & Keskinocak, P. (2003). Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management Science, 49*(10), 1287-1309.

Federgruen, A., & Heching, A. (1999). Combined pricing and inventory control under uncertainty. *Operations Research,* 47(3), 454-475.

Feng, Y., & Gallego, G. (1995). Optimal starting times for end-of-season sales and optimal stopping times for promotional fares. *Management Science, 41*(8), 1371-1391.

Huang, S., Yang, Y., & Anderson, K. (2001). A theory of finitely durable goods monopoly with used-goods market and transaction cost. *Management Science, 47*(11), 1515-1532.

Jeffrey, I. M., & Garrett, J. V. R. (1999). Revenue management: Research overview and prospects. *Transportation Science, 33*(2), 233-257.

Kadet, A. (2004). The price is right. *Smart Money, 12*(3), 90-94.

Liu, Q., & van Ryzin, G. (2011). Strategic capacity rationing when customers learn. *Manufacturing Service Operations Management, 13*(1), 89-107.

Liu, Q., & Zhang, D. (2013). Dynamic pricing competition with the strategic customers under the vertical product differentiation. *Management Science, 59*(1), 84-101.

Marcus, B., & Anderson, C. (2008). Revenue management for low-cost providers. *European Journal of Operational Research, 188*(1), 258-272.

Preyas, S. D., & Devavrat, P. (1999). Competition in durable goods markets: The strategic consequences of leasing and selling. *Marketing Science,* 18(1), 42-58.