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Spurious Relationship of AR(P) Stable Sequences in Presence of Trends Breaks

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Abstract
This paper analyzes spurious regression phenomenon involving AR(p) stable processes with trend breaks. It shows that when those time series are used in ordinary least squares regression, the convenient t-ratios procedures wrongly indicate that the spurious relationship is present as the pair of independent stable series contains trend changes. The spurious relationship becomes stronger as the sample size approaches to infinite. As a result, spurious effects might occur more often than we previously believed as they can arise even between AR(p) stable series in present of trend breaks.

Key words: Spurious relationship; Stable sequence; T-ratios; Trend breaks

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INTRODUCTION

Spurious regression is a situation in which two or more variables are statistically related, but in fact there is not any direct relation between them. It is conceived in the time series econometric literature, can be traced back to Yule (1926), who identified the phenomenon by means of a computerless Monte Carlo experience in which correlation coefficients were obtained from pairs of independent non-stationary variables. Granger and Newbold (1974) identified it again for simple least squares estimates and showed that when unrelated data series are close to the integrated processes of order one or the I(1) processes, then running a regression with this type of data will yield spurious effects. Phillips (1986) provided the theoretical framework to understand the phenomenon in the simplest case(independent driftless unit root processes (DGPs)), such as unit root with drifts, trend stationary and long range.

The above results served as a springboard to a subsequent long series of investigations of the phenomenon for different types of regression and different types of data generation process. Marmol (1998) suggested that spurious correlation generally occurs in regressions including fractionally integrated processes. Spurious regressions are also shown to occur in models with series generated by various combinations of different types of stationary processes by Granger et al. (2001). For more details about spurious regression we refer the reader to Lizeth and Daniel (2011) and Martinez-Rivera and Ventosa-Santaularia (2012), among many others.

All these studies rely on the case where variances of the sequence are finite and, therefore, demonstrate the existence of spurious regression under finite-variance. However, there is growing body of evidence showing that many economic and financial time series have volatilities that are stable sequence with infinite-variance. Many types of data from economics and finance have the same character: a heavier tail than the normal variables, and it is more suitable to model these heavy-tailed data by some processes belonging to the domain of attraction of a stable law with stable index, where the stable index can reflect the heaviness of the data. This kind of data was considered by Tasy (1999) and Phillips (1990). Later on, Rechev and Mittnik (2000) and Kokoszka and Taqqu (2001) studied linear processes with heavy-tailed distributions; Davis and Mikosch (1998) and

McElroy and Politis (2002) have developed the asymptotic theory for sample autocovariances and extreme for such processes. The purpose of this paper is to investigate the asymptotic behavior of the usual diagnostic statistics when they are employed to test if there exists a relationship between two independent stable sequences with infinite-variance in presence of trend breaks. Thus, this paper is to extend the interval of tailed index from k=2(Gaussian series) to $k\in(1,2]$(Infinite-variance sequences).

This paper is organized as follows. In Section 2, we present the data-generation processes with structural breaks in trend and the assumptions made on the various components. Section 3 deals with the asymptotic properties of the least squares estimates involving trend breaks. In Section 4, we would provide some simulation evidence, whilst conclusions are drawn in Section 5.

1. THE MODELS AND ASSUMPTIONS

Our analysis of the spurious effects are based on simple regression models where the dependent variable and the single nonconstant regressor are independent infinite-variance processes with structural breaks in mean. Before presenting these models, let us first briefly review some basic properties of the stable process.

We consider moving average of the form

$$y_t = C(L)\varepsilon_t = \sum_{j=0}^{\infty} c_j \varepsilon_{t-j} \quad (1)$$

where $y_t = C(L)\varepsilon_t = \sum_{j=0}^{\infty} c_j \varepsilon_{t-j}$ and the weights c_j satisfying

$$\sum_{j=0}^{\infty} |c_j| < \infty \quad (2)$$

This model nests causal $ARMA(p,q)$ and $AR(\infty)$ specifications. The independent identical distribution innovations ε_t are assumed to be mean zero and in the domain of attraction of a stable law with 1<k<2. Thus ε_t satisfy the following assumption:

Assumption 2.1 The innovations ε_t are in the domain of attraction of a stable law with tailed index $k\in(1,2)$ and $E\varepsilon_t$=0.

Our method also relies on the results derived by Resnick (1987).

Lemma 2.1 If Assumption 2.1 holds, then

$$\left(a_T^{-1}\sum_{t=1}^{[Tr]}\varepsilon_t, a_T^{-2}\sum_{t=1}^{[Tr]}\varepsilon_t^2\right) \xrightarrow{d} \left(Z(r), W^2(r)\right),$$

where

$$a_T = \inf\left\{x : P\left(|\varepsilon_t| > x\right) \le T^{-1}\right\},$$

and the random variable $Z(r)$ is k— stable and $W_2(2)$ is k/2—stable *Levy process* in [0,1]. The notation $\xrightarrow{d}$ stands for convergence in distribution.

The exact definition of the Levy process (Z(r),$W^2(r)$) appearing in Lemma 2.1 is not needed in the following, but we recall that the quantities a_T can be represented as

$$a_T = T^{1/\kappa} J(T),$$

for some slowly varying function J.

Lemma 2.2 Suppose y_t are defined by (1). If Assumption 2.1 and (2) hold, then

$$\left(a_T^{-1}\sum_{t=1}^{T} y_t,\ a_T^{-2}\sum_{t=1}^{T} y_t^2,\ a_T^{-2}\sum_{t=s+1}^{T} y_t y_{t-s}\right)$$

$$\xrightarrow{d}\left(\left(\sum_{j=0}^{\infty} c_j\right)Z(1),\ \left(\sum_{j=0}^{\infty} c_j^2\right)W^2(1),\ \left(\sum_{j=0}^{\infty} c_j c_{s+j}\right)W^2(1)\right),$$

where $Z(1)$,$W^2(1)$ and a_T are defined in Lemma 2.1.

Interestingly, Lemma 2.2 does not extended directly to a functional version, as it does in Lemma 2.1. This has been discovered by Avram and Taqqu (1986) and Resnick (1987). However, they still proved the results that $\sum_{t=1}^{[Tr]} y_t = Op(a_T)$ and $\sum_{t=1}^{[Tr]} y_T^2 = Op(a_T^2)$, which are necessary to derive the asymptotic validity of our test procedures.

To examine the spurious effects, we first define two independent series u_t and v_t, which satisfy Assumption 2.1 with tailed indices k_u and k_v, respectively. Now, we consider two stable processes x_t (explanation variable) and y_t (depedent variable) generated from the following DGP:

$$A(L)x_t = \mu_x + \theta_x t + \gamma_x (t - [T\tau_x])1_{\{t>[T\tau_x]\}} + u_t, \quad (3)$$

$$B(L)y_t = \mu_y + \theta_y t + \gamma_y (t - [T\tau_y])1_{\{t>[T\tau_y]\}} + v_t, \quad (4)$$

where lag polynomials, $A(L)$and $B(L)$, have their roots lying outside the unit circle; x_t is the explanation variable and y_t is the depedent variable; μ_x and μ_y are the intercept of x_t and y_t; θ_x and γ_x are, respectively, the permanent trend and the transitory trend, resulting from a break, of the process x_t; θ_y and γ_y are, respectively, the permanent trend and the transitory trend, resulting from a break, of the process y_t. Both $1_{\{t>[T\tau_x]\}}$ and $1_{\{t>[T\tau_y]\}}$ equal 1 when $t>[T\tau_x]$ and $t>[T\tau_y]$, or equal 0. The below Theorem 3.1 indicates that (3) and (4) exist the

spurious regression, but in fact there is not any direct relation between them.

Let us define the following inverse lag operators: $\bar{A}(L) = A^{-1}(L) = \sum_{j=0}^{\infty} a_j L^j$ and

$\bar{B}(L) = B^{-1}(L) = \sum_{j=0}^{\infty} b_j L^j$, with $\sum_{j=0}^{\infty}|a_j| < \infty$ and $\sum_{j=0}^{\infty}|b_j| < \infty$. Because the roots of $A(L)$ and $B(L)$ are outside the unit circle. Therefore, it follows from the BN (Beveridge and nelson) decomposition can be used as following, yields

$$\bar{A}(L) = \bar{A}(1) + (1-L)\tilde{A}(L),$$
$$\bar{B}(L) = \bar{B}(1) + (1-L)\tilde{B}(L),$$

Where $\tilde{A}(L) = \sum_{j=0}^{\infty}\tilde{a}_j, \tilde{a}_j = \sum_{k=j+1}^{\infty} a_k$ and $\tilde{B}(L) = \sum_{j=0}^{\infty}\tilde{b}_j, \tilde{b}_j = \sum_{k=j+1}^{\infty} b_k$. BN decomposition yields directly the martingale approximation to the partial sum process of a stationary time series, see Hall and Heyde(1980).

We assume, without loss of generality, that the initial values of the stable processes u_0, v_0, x_0 and y_0 are all zero. Hence, x_t and y_t can be rewritten as

$$x_t = \overline{A}(1)\mu_x + \theta_x(\overline{A}(1)t + \overline{A}(1)) + \gamma_x \left\{\overline{A}(1)(t - [T\tau_x]) + \overline{A}(1)\right\} 1_{\{t>[T\tau_x]\}} + \overline{A}(L)u_t, \tag{5}$$

$$y_t = \overline{B}(1)\mu_y + \theta_y(\overline{B}(1)t + \overline{B}(1)) + \gamma_y \left\{\overline{B}(1)(t - [T\tau_y]) + \overline{B}(1)\right\} 1_{\{t>[T\tau_y]\}} + \overline{B}(L)v_t, \tag{6}$$

In the view of Lemma 2.1 and 2.2, if we define $a_{u,T} = T^{1/\kappa_u} J_u(T)$ and $a_{v,T} = T^{1/\kappa_v} J_v(T)$,

then we have

$$\left(a_{u,T}^{-1}\sum_{t=1}^{T}\overline{A}(L)u_t, a_{v,T}^{-1}\sum_{t=1}^{T}\overline{B}(L)v_t\right) \xrightarrow{d} \left(\left(\sum_{j=0}^{\infty}a_j\right)Z_u(1), \left(\sum_{j=0}^{\infty}b_j\right)Z_v(1)\right),$$

and

$$\left(a_{u,T}^{-2}\sum_{t=1}^{T}\overline{A}^2(L)u_t^2, a_{v,T}^{-2}\sum_{t=1}^{T}\overline{B}^2(L)v_t^2\right) \xrightarrow{d} \left(\left(\sum_{j=0}^{\infty}a_j^2\right)W_u^2(1), \left(\sum_{j=0}^{\infty}b_j^2\right)W_v^2(1)\right).$$

2. MAIN RESULTS

Given the preceding discussion, we consider the

$$y_t = \alpha + \delta t + \beta x_t + \xi_t \tag{7}$$

Let $\hat{\alpha}$, $\hat{\delta}$ and $\hat{\beta}$ denote the ordinary least square estimates from a regression of y_t on a constant, the trend t and x_t respectively. Their respective '*variance*' are estimated by $s_{\hat{\alpha}}^2$, $s_{\hat{\delta}}^2$ and $s_{\hat{\beta}}^2$ from which we have the diagnostic statistics $t_{\hat{\alpha}} = \hat{\alpha}/s_{\alpha}$, $t_{\hat{\delta}} = \hat{\delta}/s_{\delta}$ and $t_{\hat{\beta}} = \hat{\beta}/s_{\beta}$.

In order to determine the limit behavior of the t-ratios, the following Lemma is needed.

Lemma3.1 Suppose that (x_t, y_t) is generated by (5) and (6). The sequence, u_t and v_t, are independent and satisfy Assumption 2.1. Then, as $T \to \infty$,

1.
$$T^{-2}\sum_{t=1}^{T}t \to \int_0^1 rdr \equiv H_t, \qquad T^{-3}\sum_{t=1}^{T}t^2 \to \int_0^1 r^2dr \equiv H_{tt}.$$

2.
$$T^{-2}\sum_{t=1}^{T}x_t \Rightarrow \overline{A}\{\theta_x\int_0^1 rdr + \gamma_x\int_0^1 (r-\tau_x)dr\} \equiv H_x.$$

3.
$$T^{-3}\sum_{t=1}^{T}x_t^2 \Rightarrow \overline{A}^2(1)\left\{\theta_x^2\int_0^1 r^2dr + \gamma_x^2\int_{\tau_x}^1 (r-\tau_x)^2 dr + 2\theta_x\gamma_x\int_{\tau_x}^1 r(r-\tau_x)dr\right\} \equiv H_{xx}.$$

4.
$$T^{-3}\sum_{t=1}^{T}tx_t \Rightarrow \overline{A}(1)\left\{\theta_x\int_0^1 r^2dr + \int_{\tau_x}^1 \gamma_x r(r-\tau_x)dr\right\} \equiv H_{tx}.$$

5.
$$T^{-3}\sum_{t=1}^{T}y_t \Rightarrow \overline{B}(1)\left\{\theta_y\int_0^1 rdr + \gamma_y\int_{\tau_y}^1 (r-\tau_x)dr\right\} \equiv H_y.$$

6.
$$T^{-3}\sum_{t=1}^{T}y_t^2 \Rightarrow \overline{B}^2(1)\left\{\theta_y^2\int_0^1 r^2dr + \gamma_y^2\int_{\tau_y}^1 (r-\tau_y)^2 dr + 2\theta_y\gamma_y\int_{\tau_x}^1 r(r-\tau_y)dr\right\} \equiv H_{yy}.$$

7.
$$T^{-3}\sum_{t=1}^{T}ty_t \Rightarrow \overline{B}(1)\left\{\theta_y\int_0^1 r^2dr + \int_{\tau_y}^1 \gamma_y r(r-\tau_y)dr\right\} \equiv H_{ty}.$$

8.
$$T^{-3}\sum_{t=1}^{T}x_ty_t \Rightarrow \overline{A}(1)\overline{B}(1)\left\{\theta_x\theta_y\int_0^1 r^2dr + \gamma_x\gamma_y\int_{\max(\tau_x,\tau_y)}^1 (r-\tau_x)(r-\tau_y)dr + \theta_x\gamma_y\int_{\tau_y}^1 r(r-\tau_y)dr + \theta_y\gamma_x\int_{\tau_x}^1 r(r-\tau_x)dr\right\} \equiv H_{xy}.$$

Moreover, the items 1-8 hold irrespective of the initial conditions assigned to x_0 and y_0.

Proof The proof of item 1 of Lemma 3.1 can be available with some preliminary algebra. Now, we shall prove item 2 of Lemma 3.1. Note that

$$\begin{aligned} &T^{-2}\sum_{t=1}^{T}x_t \\ &= T^{-2}\sum_{t=1}^{T}\left(\overline{A}(1)\mu_x + \theta_x\tilde{A}(1)\right) + \theta_x\overline{A}(1)T^{-2}\sum_{t=1}^{T}t + \gamma_x\tilde{A}(1)T^{-2}\sum_{t=1}^{T}1\{t>[T\tau_x]\} \\ &\quad + \gamma_x\overline{A}(1)T^{-2}\sum_{t=1}^{T}(t-[T\tau_x])1\{t>[T\tau_x]\} + T^{-2}\sum_{t=1}^{T}\overline{A}(L)u_t \\ &= I + II + III + IV + V. \end{aligned}$$

One can verify that $I \to 0$ and $III \to 0$. In the view of Lemma 2.2 and item 1 of Lemma 3.1, yields $II \to \theta_x\tilde{A}(1)\int_0^1 rdr$, $IV \to \gamma_x\tilde{A}(1)\int_{\tau_x}^1 (r-\tau_x)dr$, and $V=Op\left(T^{-2}\cdot\alpha_{u,T}\right)=o_p(1)$.

To avoid any cumbersome mathematical expression, we shall summarize only the terms of order $O(T^{-1})$ as follows. Note also that

$$T^{-3}\sum_{t=1}^{T}x_t^2=\theta_x^2\bar{A}^2(1)T^{-3}\sum_{t=1}^{T}t^2+\gamma_x^2\bar{A}^2(1)T^{-3}\sum_{t=1}^{T}\left(t-[T\tau_x]\right)^2 1\{t>[T\tau_x]\}=I+II+III+IV.$$

One can verify that $I\to\theta_x^2\bar{A}^2(1)\int_0^1 r^2dr, II\to\gamma_x^2\bar{A}^2(1)\int_{\tau_x}^1(r-\tau_x)^2dr$ and $III\to 2\bar{A}^2(1)\theta_x\gamma_x\int_{\tau_x}^1 r(r-\tau_x)dr$. In view of Lemma 2.2, yields $IV=O_p\left(T^{-3}\cdot\alpha_{u,T}^2\right)=o_p(1)$. Hence the proof of item 3 in Lemma 2.3 is completed.

To prove item 4 of Lemma 3.1, we have

$$T^{-3}\sum_{t=1}^{T}tx_t=\theta_x\bar{A}(1)T^{-3}\sum_{t=1}^{T}t^2+\gamma_x\bar{A}(1)T^{-3}\sum_{t=1}^{T}t\left(t-[T\tau_x]\right)1\{t>[T\tau_x]\}+T^{-3}\sum_{t=1}^{T}t\bar{A}(L)u_t+\gamma_x\tilde{A}(1)T^{-3}\sum_{t=1}^{T}t1\{t-[T\tau_x]\}+O_p\left(T^{-1}\right)=I+II+III+IV+V.$$

One can see that $I\to\theta_x\bar{A}(1)\int_0^1 r^2dr$, $II\to\gamma_x\bar{A}(1)\int_{\gamma_x}^1 r(r-\tau_x)dr$ and $IV\to 0$. An application of Lemma 2.3 yields $III\to O_p\left(T^{-2}\cdot\alpha_{u,T}\right)=o_p(1)$.

Now it remains to prove item 8 of Lemma 3.1. One can verify the negligibility of items 5,6,7,9 and 10 in a similar fashion to complete the proof of Lemma 3.1. Note that

$$T^{-3}\sum_{t=1}^{T}x_ty_t=T^{-3}\sum_{t=1}^{T}\left[\left(\theta_x\bar{A}(1)t+\gamma_x\bar{A}(1)\left(t-[T\tau_x]\right)1\{t>[T\tau_x]\}\right)\left(\theta_y\bar{B}(1)t+\gamma_y\bar{B}(1)\left(t-[T\tau_y]\right)1\{t>[T\tau_y]\}\right)\right]+T^{-3}\sum_{t=1}^{T}\bar{A}(L)\bar{B}(L)u_tv_t+O_p\left(T^{-1}\right)=I+II.$$

One can see that $II\Rightarrow 0$ and

$$\begin{aligned}I&\Rightarrow\int_0^1\left(\theta_x\bar{A}(1)r+\gamma_x\bar{A}(1)(r-\tau_x)1\{r>\tau_x\}\right)\left(\theta_y\bar{B}(1)r+\gamma_y\bar{B}(1)(r-\tau_y)1\{r>\tau_y\}\right)dr\\&=\bar{A}(1)\bar{B}(1)\Big\{\theta_x\theta_y\int_0^1 r^2dr+\theta_x\gamma_y\int_0^1 r(r-\gamma_y)dr+\theta_y\gamma_x\int_{\tau_x}^1 r(r-\gamma_x)dr\\&\quad+\gamma_x\gamma_y\int_{\max(\tau_x,\tau_y)}^1(r-\tau_x)(r-\tau_y)dr\Big\}.\end{aligned}$$

Let's define

$$Q=\begin{bmatrix}1&H_t&H_x\\H_t&H_{tt}&H_{tx}\\H_x&H_{tx}&H_{xx}\end{bmatrix}\text{ and }P=\begin{bmatrix}H_y\\H_{ty}\\H_{xy}\end{bmatrix}$$

where the elements of the matrices are defined in Lemma 3.1.

The limiting behavior of regression statistics are stated in Theorem 3.1.

Theorem 3.1 Suppose that the conditions of Lemma 3.1 are satisfied. Then, as $T\to\infty$,

a) $$\begin{bmatrix}T^{-1}\hat{\alpha}\\\hat{\delta}\\\hat{\beta}\end{bmatrix}\Rightarrow Q^{-1}P=\begin{bmatrix}H_{\hat{\alpha}}\\H_{\hat{\delta}}\\H_{\hat{\beta}}\end{bmatrix},$$

b) $$\begin{bmatrix}T^{-1/2}t_{\hat{\alpha}}\\T^{-1/2}t_{\hat{\delta}}\\T^{-1/2}t_{\hat{\beta}}\end{bmatrix}\Rightarrow\begin{bmatrix}H_{\hat{\alpha}}/H_{s_{\hat{\alpha}}}\\H_{\delta}/H_{s_{\hat{\delta}}}\\H_{\hat{\beta}}/H_{s_{\hat{\beta}}}\end{bmatrix},$$

where

$$H_{s_{\hat{\alpha}}}=H_{ss}^{1/2}I_{(1)}Q^{-1}I_{(1)}^{'}$$
$$H_{s_{\hat{\delta}}}=H_{ss}^{1/2}I_{(2)}Q^{-1}I_{(2)}^{'}$$
$$H_{s_{\hat{\beta}}}=H_{ss}^{1/2}I_{(3)}Q^{-1}I_{(3)}^{'}$$

$$H_{ss}=H_{yy}+H_{\hat{\alpha}}^2+H_{\beta}^2H_{xx}+H_{\delta}^2H_{tt}+2H_{\alpha}H_{\delta}H_t+2H_{\alpha}H_{\beta}H_x+2H_{\hat{\delta}}H_{\beta}H_{tx}-2H_{\alpha}H_y-2H_{\delta}H_{ty}-2H_{\beta}H_{xy}.$$

And $I(i)$denotes a vector in which the i-th element is one and other elements are zeros.

Proof of Theorem 3.1 Using the results from Lemma 3.1, we have

$$\begin{bmatrix} T^{-3}\hat{\alpha} \\ \hat{\delta} \\ \hat{\beta} \end{bmatrix} = \begin{bmatrix} T & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{\alpha} \\ \delta \\ \beta \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & T^{-1} & 0 \\ 0 & 0 & T^{-1} \end{bmatrix} \begin{bmatrix} T & \sum_{t=1}^{T} t & \sum_{t=1}^{T} x_t \\ \sum_{t=1}^{T} t & \sum_{t=1}^{T} t^2 & \sum_{t=1}^{T} tx_t \\ \sum_{t=1}^{T} x_t & \sum_{t=1}^{T} tx_t & \sum_{t=1}^{T} x_t^2 \end{bmatrix} \begin{bmatrix} T^{-1} & 0 & 0 \\ 0 & T^{-2} & 0 \\ 0 & 0 & T^{-2} \end{bmatrix} \right\}^{-1}$$

$$\times \begin{bmatrix} T^{-2} & 0 & 0 \\ 0 & T^{-3} & 0 \\ 0 & 0 & T^{-3} \end{bmatrix} \begin{bmatrix} \sum_{t=1}^{T} y_t \\ \sum_{t=1}^{T} x_t y_t \\ \sum_{t=1}^{T} ty_t \end{bmatrix}$$

$$= \begin{bmatrix} 1 & T^{-2}\sum_{t=1}^{T} t & T^{-2}\sum_{t=1}^{T} x_t \\ T^{-2}\sum_{t=1}^{T} t & T^{-3}\sum_{t=1}^{T} t^2 & T^{-3}\sum_{t=1}^{T} tx_t \\ T^{-2}\sum_{t=1}^{T} x_t & T^{-3}\sum_{t=1}^{T} tx_t & T^{-3}\sum_{t=1}^{T} x_t^2 \end{bmatrix}^{-1} \begin{bmatrix} T^{-2}\sum_{t=1}^{T} x_t \\ T^{-3}\sum_{t=1}^{T} ty_t \\ T^{-3}\sum_{t=1}^{T} x_t y_t \end{bmatrix}$$

$$\Rightarrow Q^{-1}P.$$

Moreover, we have

$$T^{-2}s_{\hat{\alpha}} = \left\{T^{-3}\hat{s}^2\right\} II_{(1)} \begin{bmatrix} 1 & T^{-2}\sum_{t=1}^{T} t & T^{-2}\sum_{t=1}^{T} x_t \\ T^{-2}\sum_{t=1}^{T} t & T^{-3}\sum_{t=1}^{T} t^2 & T^{-3}\sum_{t=1}^{T} tx_t \\ T^{-2}\sum_{t=1}^{T} x_t & T^{-3}\sum_{t=1}^{T} tx_t & T^{-3}\sum_{t=1}^{T} x_t^2 \end{bmatrix} II_{(1)}^{'},$$

$$Ts_{\hat{\delta}} = \left\{T^{-2}\hat{s}^2\right\} II_{(2)} \begin{bmatrix} 1 & T^{-2}\sum_{t=1}^{T} t & T^{-2}\sum_{t=1}^{T} x_t \\ T^{-2}\sum_{t=1}^{T} t & T^{-3}\sum_{t=1}^{T} t^2 & T^{-3}\sum_{t=1}^{T} tx_t \\ T^{-2}\sum_{t=1}^{T} x_t & T^{-3}\sum_{t=1}^{T} tx_t & T^{-3}\sum_{t=1}^{T} x_t^2 \end{bmatrix} II_{(2)}^{'},$$

$$Ts_{\hat{\beta}} = \left\{T^{-2}\hat{s}^2\right\} II_{(3)} \begin{bmatrix} 1 & T^{-2}\sum_{t=1}^{T} t & T^{-2}\sum_{t=1}^{T} x_t \\ T^{-2}\sum_{t=1}^{T} t & T^{-3}\sum_{t=1}^{T} t^2 & T^{-3}\sum_{t=1}^{T} tx_t \\ T^{-2}\sum_{t=1}^{T} x_t & T^{-3}\sum_{t=1}^{T} tx_t & T^{-3}\sum_{t=1}^{T} x_t^2 \end{bmatrix} II_{(3)}^{'},$$

$$\begin{aligned} T^{-2}\hat{s}^2 &= T^{-3}\sum_{t=1}^{T} \hat{\xi}_t^2 \\ &= T^{-3}\sum_{t=1}^{T} \left\{ y_t^2 + \hat{\beta}^2 x_t^2 + \hat{\alpha}^2 + \delta^2 t^2 - 2\alpha y_t - 2\delta t y_t - 2\beta x_t y_t + 2\alpha\delta t + 2\alpha\beta x_t + 2\delta\beta t x_t \right\} \\ &\Rightarrow H_{yy} + H_{\hat{\beta}}^2 H_{xx} + H_{\alpha}^2 + H_{\delta}^2 H_{tt} + 2H_{\alpha} H_{\delta} H_t + 2H_{\alpha} H_{\beta} H_x + 2H_{\delta} H_{\beta} H_{tx} - 2H_{\alpha} H_y \\ &- 2H_{\hat{\delta}} H_{ty} - 2H_{\beta} H_{xy} \equiv H_{ss}, \end{aligned}$$

and

$$\begin{aligned} &T^{-3}\sum_{t=2}^{T} \hat{\xi}_{t-1}\xi_t \\ &= T^{-3}\sum_{t=2}^{T} \Big\{ y_{t-1}y_t + \hat{\beta}^2 x_{t-1}x_t + \hat{\alpha}^2 + \delta^2 (t-1)t - \alpha(y_{t-1} + y_t) - \delta\left(ty_{t-1} + (t-1)y_t\right) \\ &\quad - \hat{\beta}(y_{t-1}x_t + x_{t-1}y_t) + \hat{\alpha}\delta(2t-1) + \alpha\beta(x_{t-1} + x_t) + \delta\beta\left((t-1)x_t + tx_{t-1}\right) \Big\} \\ &\Rightarrow H_{ss}. \end{aligned}$$

By applying Lemma 3 .1, one can immediately derive the limits of individual terms in the above equations. Theorem 3.1 has been proved.

3. SIMULATION

In this section we use Monte Carlo simulation methods to examine the sample performance of our theoretical results in Section 2 and 3. We compute rejection frequency of thet-ratios for testing the null hypotheses H_0:β=0, in equations (7). All results are obtained by 3000 replications using a 1.96 critical value (5% level) for a standard normal distribution.

We consider the properties of the t-ratios when the data-generating processes exhibit structural breaks in trend. For the simulation, we let

$$x_t = \phi_x x_{t-1} + \mu_x + \theta_x t + \gamma_x (t - [T\tau_x])1_{\{t>[T\tau_x]\}} + u_t,$$

$$y_t = \phi_y y_{t-1} + \mu_y + \theta_y t + \gamma_y (t - [T\tau_y])1_{\{t>[T\tau_y]\}} + v_t,$$

Where $\mu_x = 0.8, \mu_y = 1.2, \theta_x = 0.3, \theta_y = 0.2, \gamma_x = 0.1$ and γ_y=0.2 The values of autoregressive parameters are still chosen to be$\{0.0,0.2,0.5,0.8,1.0\}$. The innovation processes u_t and v_t. The spurious regression of generated by the program of STABLE are independent of each other and satisfy Assumption 2.1 with tailed indexes $k_u$and k_v varying among $\{1.2,1.3,1.8\}$. Moreover, the program STABLE is available from J. P. Nolans website: academic2.american.edu/.jpnolan. We just report the results for ϕ=ϕ_x=ϕ_y, and the other cases have similar results.

Table1
Regressing Between Two AR(1) Infinite-Variance Series With Breaks in Trendϕ=ϕ_x=ϕ_y), Rejection frequency of $|t_\beta| > 1.96$.

		k_1=1.3, k_2=1.2					k_1=1.3, k_2=1.8				
τ_x,τ_y	T/ϕ	0.0	0.2	0.5	0.8	1.0	0.0	0.2	0.5	0.8	1.0
0.10 0.10	500	20.63	30.55	59.62	84.50	100	88.00	90.48	96.43	97.82	100
	1000	65.12	72.52	85.51	94.50	100	96.42	98.00	99.00	99.24	100
	10000	99.10	99.43	99.30	99.91	100	100	100	100	100	100
0.10 0.90	500	2.20	5.50	14.87	39.83	100	18.31	24.42	45.11	69.10	100
	1000	9.44	15.26	33.76	58.82	100	66.35	74.31	83.32	91.27	100
	10000	95.00	96.00	97.89	99.10	100	100	100	100	100	100
0.50 0.50	500	90.35	93.50	95.77	98.15	100	98.12	99.00	99.47	99.81	100
	1000	97.11	97.00	98.80	99.22	100	99.47	100	100	100	100
	10000	99.13	99.17	99.33	99.76	100	100	100	100	100	100
0.90 0.10	500	3.58	5.00	12.95	32.45	100	5.64	6.16	12.76	30.67	100
	1000	5.97	10.41	26.20	51.11	100	51.36	60.75	76.49	85.73	100
	10000	94.89	97.78	97.32	99.00	100	100	100	100	100	100
0.90 0.90	500	26.54	36.50	62.00	78.73	100	89.23	92.66	94.86	97.45	100
	1000	64.45	74.60	86.06	92.23	100	98.06	97.62	99.28	99.40	100
	10000	99.00	99.39	99.74	99.67	100	100	100	100	100	100

Table 1 report the simulated empirical power for the case of structural breaks in trend. There are some conclusions should be mentioned. Firstly, we will find the phenomenon of spurious regression driven by trend breaks is serious, since the magnitude of the probability limit of $t_{\hat\beta}$ increases even further. Hence, the rejection rate of 100% is wel predicted. Second, as T increases, or either ϕ_x or ϕ_y increass, the rejection powe increases. The rejection rate for the T=1000, $\tau_x = 0.1, \tau_y = 0.9, \phi_x = 0.8, \phi_y = 0.8$ are 58.82% for k_1=1.3,k_2=1.2, and 91.27% for k_1=1.3,k_2=1.8, confirming the consistency results of Theroem 3.1. Finally, it is clear that the less tailed indexes provide a lower empirical power. It is mainly because both DGP x_t and y_t have more 'outliers' when the tailed indexes decrease. This conclusion that the test statistics are sensitive to the tailed index, and the similar results can been seen in the paper of Jin et al. (2009).

In order to give further intuitive idea for the influence of autoregressive parameter, break fraction and tailed index, we provide the rejection frequency of two pairs of tailed indexes with $\phi = 0, 0.2, 0.5, 1, \tau_x, \tau_y \in (0,1)$ and sample size T=1000in Figure 1-2.

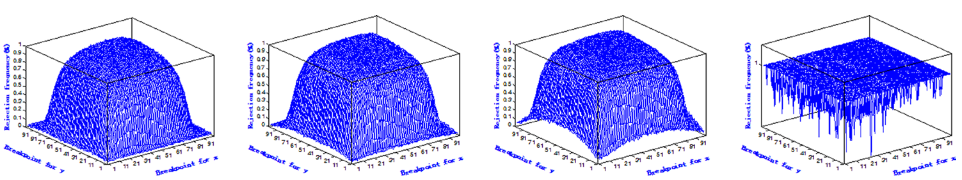

Figure 1
The First Panel Represents Rejection Frequency For Autoregressive Parameters $\phi = 0$ and the Last Three for $\phi = 0.2$, $\phi = 0.5$ and $\phi = 1$ ($\kappa_1 = 1.3$, $\kappa_2 = 1.2$) Respectively

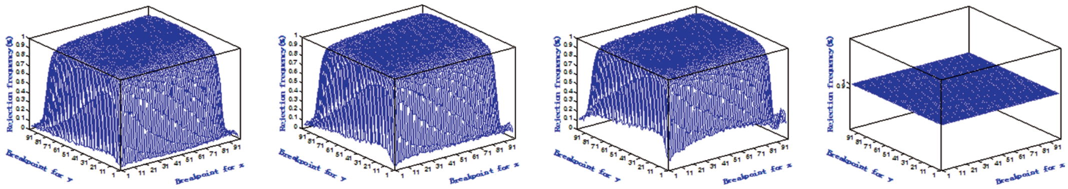

Figure 2
The First Panel Represents Rejection Frequency for Autoregressive Parameters $\phi = 0$ and the Last Three for $\phi = 0.2$, $\phi = 0.5$ and $\phi = 1$ ($\kappa_1 = 1.3$, $\kappa_2 = 1.8$) Respectively

Figure 1-2 report that the null hypothesis of $t_{\hat{\beta}}$ will almost be rejected with certainty even if $\phi = 0$ in the case of trend breaks. In a word, our simulation experiments confirm our motivation a spurious relationship is present in these regressions in the case of structural breaks in trend.

CONCLUSION

The research of Tsay (1999) examined the possibility of spurious relationship between two independent integrated errors processes belonging to the domain of attraction of a stable law with tailed index κ, and showed that the *t*-ratios diverge at the rate, which is identical to what Phillips (1986) has obtained for the Gaussian case where $\kappa = 2$. This paper has been extended to consider the properties of t-ratios allowing for structural breaks in the regressive relationship when applied to independent infinite-variance series subject to breaks in trend. We find that using this fairly standard AR(p) framework allows us to successfully address the questions whether spurious regctural breaks in trend is found to be ($T^{1/2}$). A fairly extensive Monte Carlo study has also been conducted to verify the performance of our test procedures, especially those of convergence rate established in the paper. Hence, it is likely to find a spuriously statistical significant relationship between two independent stable AR(p) processes subject to trend breaks when the regression model includes a linear trend in its deterministic specification.

REFERENCES

Avram, F., & Taqqu, M. S. (1986). Weak convergence of moving average with infinite variance. *In Dependence in Probability and Statistics*, 137-162.

Davis, R. A., & Mikosch, T. (1998). Limit theory for the sample acf of stationary process with heavy tails with applications to ARCH. *The Annals of Statistics*, *26*, 2049-2080.

Granger, C. W. J., Hyung, N., & Jeon, Y. (2001). Spurious regressions with stationary series. *Applied Economics*, *33*, 899-904.

Granger, C. W. J., & Newbold, P. (1974). Spurious regressions in econometrics. *Journal of Econometrics*, *2*, 111-120.

Hall, P., & Heyde, C. C. (1980). *Martingale limit theory and its applications*. New York: Academic.

Jin, H., Tian, Z., & Qin, R. (2009). Bootstrap tests for structural change with infinite variance observations. *Statistics and Probability Letters*, *79*, 1985-1995.

Kokosza, P., & Taqqu M. S. (2001). Can one use the Durbin-Levinson algorithm to generate infinite variance fractional ARIMA time series? *Journal of Times Series Analysis*, *22*, 317-337.

Lizeth, G. B., & Daniel, V. S. (2011). Spurious regression and lurking variables. *Statistics & Probability Letters*, *81*(12), 2004-2011.

Marmol, F. (1998). Spurious regression theory with nonstationary fractionally integrated processes. *Journal of Econometrics*, *84*, 233-250.

Martinez-Rivera, B., Ventosa-Santaulµaria, D. (2012). A comment on "Is the spurious regression problem spurious?" *Economics Letters*, *115*, 229-231.

McElroy, T., & Politis, D. D. (2002). Robust inference for the mean in the presence of serial correlation and heavy-tailed distributions. *Econometric Theory*, *18*(5), 1019-1039.

Mittnik, S., & Rachev, S. T. (2000). *Stable paretian models in finance*. New York: Wiley.

Phillips, P. C. B. (1986). Understanding spurious regressions in econometrics. *Journal of Econometrics*, *33*, 311-340.

Phillips, P. C. B. (1990). Time series regression with a unit root and infinite variance errors. *Econometric Theory*, *4*, 44-62.

Resnick, S. I. (1986). Point processes regular variation and weak convergence. *Advanced in Applied Probability*, *18*, 66-138.

Tsay, W. J. (1999). Spurious regression between I(1) processes with infinite variance errors. *Econometric Theory*, *15*, 622-628.

Yule, U. (1926). Why do we sometimes get nonsense-correlations between times series? A study in sampling and the nature of time series. J*ournal of the Royal Statistical Society*, *89*, 10-13.

